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ABSTRACT

This document was developed as a resource for secondary mathematics educators. The supplemental guide is designed to help educators challenge their students and provide each student with learning experiences to help them achieve success. Chapters reflect the mathematics curriculum and within each subject, activities are listed in the common sequence of topics. Many of the activities are designed to be adapted to other instructional areas at the discretion of the teacher. The chapters are: (1) "General Mathematics"; (2) "Algebra I"; (3) "Geometry"; (4) "Algebra II"; (5) "Trigonometry"; (6) "Math Analysis"; and (7) "Resources." An Appendix and Selected Answers section are also included. (PK)

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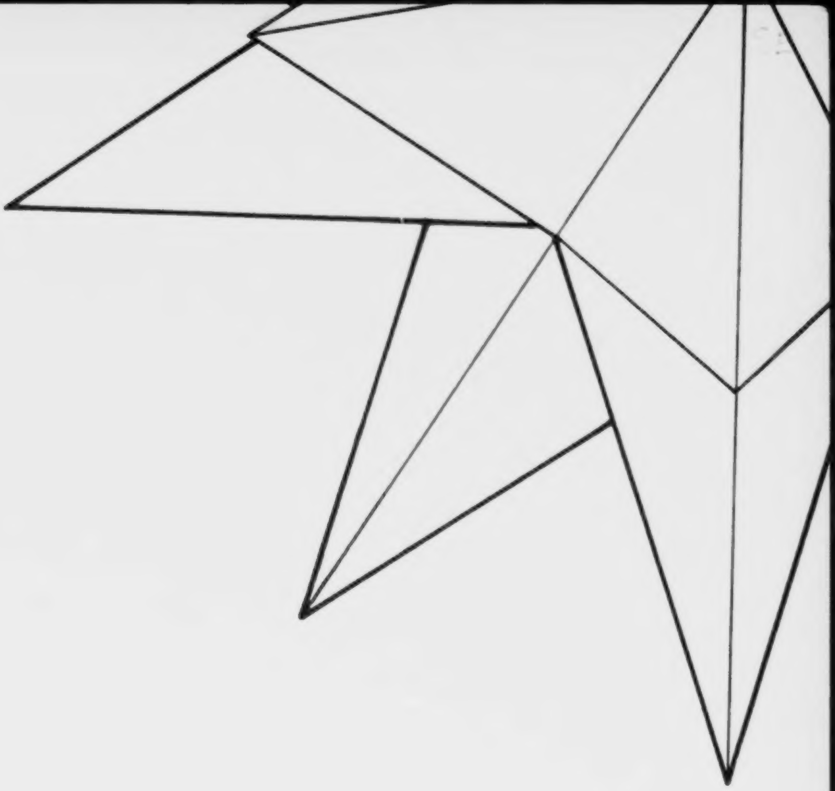

PRISM

Principal Resources In Secondary Mathematics

OKLAHOMA STATE DEPARTMENT OF EDUCATION
John M. Folks, State Superintendent
Lloyd Graham, Deputy Superintendent

PREPARED BY
Curriculum Section

Oklahoma City, Oklahoma
1985

FOREWORD

Mathematical skills and the ability to apply those skills take on increasing importance as society becomes more sophisticated in the use of technology. A necessary foundation for the job skills of tomorrow's citizens is to be found in secondary mathematics.

PRISM has been developed as an aid to Oklahoma educators who are interested in the improvement of mathematics education. Resources have been included that will assist in meeting the needs of students at all ability levels.

The State Department of Education is pleased to provide this resource to mathematics educators in Oklahoma. We gratefully acknowledge and thank all of those who contributed to the final product.

John M. Folks

John M. Folks
State Superintendent

INTRODUCTION

Today's technological explosion has plunged society into the age of information. That set of information which an individual needs to function successfully in society is constantly changing. A teacher's responsibility is to provide students with a core of basic mathematical skills and to develop their ability for original, creative thought.

Along with the daily challenges in teaching, educators acknowledge a goal to foster within students a receptive and inquisitive attitude toward solving problems. To develop confidence in their ability to solve problems, teachers must give students the opportunity to experience success using various strategies in different types of problem situations. Time-consuming calculations involved in real-life problems can be performed quickly with a calculator. Using a computer to solve a problem may involve analysis and creativity. The computer and calculator, however, will not replace the need for basic mathematical thinking skills.

Educators must continue to challenge their students and provide each with learning experiences to help them achieve success. PRISM has been developed as a resource for secondary mathematics educators to be used as a supplemental guide to help meet that goal.

Chapters reflect the mathematics curriculum and within each subject, activities are listed in the common sequence of topics. Time and space have not allowed the inclusion of activities for every concept within each subject. Many activities can be adapted to other instructional areas at the discretion of the teacher.

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CONTENTS

GENERAL MATHEMATICS 1

ALGEBRA I 35

GEOMETRY 65

ALGEBRA II 89

TRIGONOMETRY 113

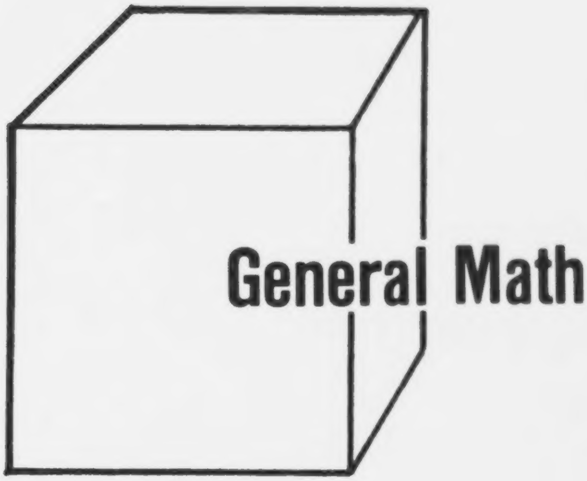
MATH ANALYSIS 145

RESOURCES 179

APPENDIX 201

SELECTED ANSWERS 227

GENERAL MATH
ADD, SUBTRACT, MULTIPLY, DIVIDE
REINFORCE THOSE SKILLS


General Math

GENERAL MATH

A secondary general mathematics course is designed to provide students with the mathematical skills they will need to function successfully in society. Operations with whole numbers, decimals, fractions and percents, as well as applications of those concepts, are included. The collection and presentation of data using charts, tables and graphs should be experienced by students. Students will also become familiar with basic geometric concepts and be able to find the perimeter, area, and volume of simple figures. Skills involved with estimating answers and the appropriate use of calculators and computers should be developed. Skills in budgeting, taxes and finance should be included to help the student become an informed consumer. Community resource persons can provide students with many real world applications of mathematics.

TOPIC	PAGE
WHOLE NUMBERS	5
DECIMALS	10
FRACTIONS	16
RATIO AND PROPORTION	21
PERCENT	23
MEASUREMENT	26
GRAPHING	32

The contents of this chapter are organized by four strands coded with symbols as follows:

✏ - Activity

🎲 - Game

🌎 - Application

🔍 - Problem-solving Experience

Many activities are coded under two or more strands.

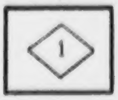
WHOLE NUMBERS

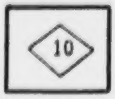
CONCEPT OBJECTIVE: The student will practice the concepts of place value.

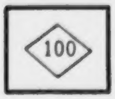
NUMBER BASE COMMUNITIES

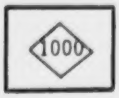
PROCEDURE: The class may be divided into small groups of 3-6 students. Each group may be considered a community and will be assigned a number base. Students may give their community a name that is indicative of their number base. For example, Quadville may represent base four.

Each group receives several sheets of one color of construction paper. Each community makes paper money bills from their colored paper. Bills of each community are labeled 1, 10, 100, and 1000. The value of a bill will vary among communities. For example, the bills in Quadville numbered 1, 10, 100, and 1000 represent one, four, sixteen, and sixty-four. Students can place a symbol on their bills that will indicate their number base. Example bills for Quadville utilize a quadrilateral symbol.









Upon completion, all groups will have money bills numbered 1, 10, 100, and 1000. However, bills will be of different colors, have different symbols, and be worth different amounts.

Students could then use the money to purchase items on a list from other communities. Purchases could also be made from a catalog with students from each community identifying what bills they would have to use to buy specific items.

WHOLE NUMBERS

CONCEPT OBJECTIVE: The student will practice operations with whole numbers.

COVER 16

PROCEDURE: Each pair of players needs a game board and one pair of 12-sided dice. Each player needs 16 markers. A player in turn rolls both dice. The player uses the two numbers on the dice and each of the four operations to determine which numbers to cover on his/her side of the game board. A player could cover as many as four numbers on one roll of the dice. For example, if a player rolled 2 and 8, the sum is 10, the difference is 6, the product is 16, and the quotient is 4. The player could cover all of these answers on his/her game board. The first player to cover all sixteen numbers on his/her side of the board is the winner.

A game board can be constructed by pasting the two game sheets on the inside of a file folder. Reproducible game sheets may be found in the appendix.

CONCEPT OBJECTIVE: The student will practice mental computation with whole numbers.

MENTAL MISSION

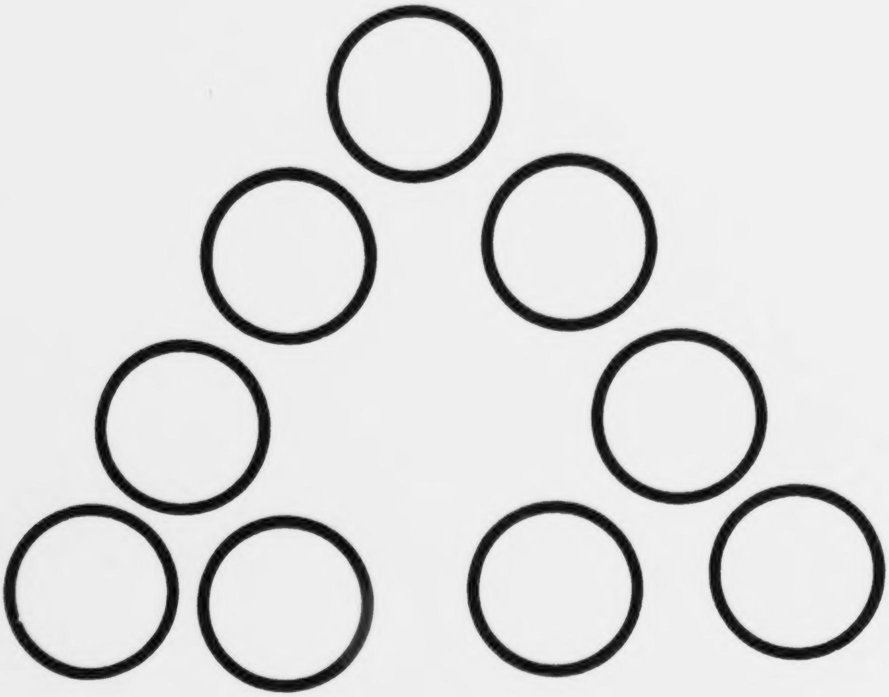
PROCEDURE: Practice in mental arithmetic skills can be accomplished in an enjoyable manner. To begin with, the teacher should state the "mental mission", a problem that is to be individually worked mentally and silently. Quick and easy combinations should be used at first, such as, "seven times nine, answer" and "one plus four plus two plus five times two, answer". The word, "answer", is the clue to the students indicating the end of the problem. The teacher should expect the solution to be given immediately. Problems may become more complex as students' skills improve. Students may create and give "mental missions" to other students.

WHOLE NUMBERS

CONCEPT OBJECTIVE: The student will practice logical thinking and adding whole numbers.

TRIANGLE OF CIRCLES

PROCEDURE: Place the digits 1 - 9 in the circles so that the sum of each side of the triangle is 20.

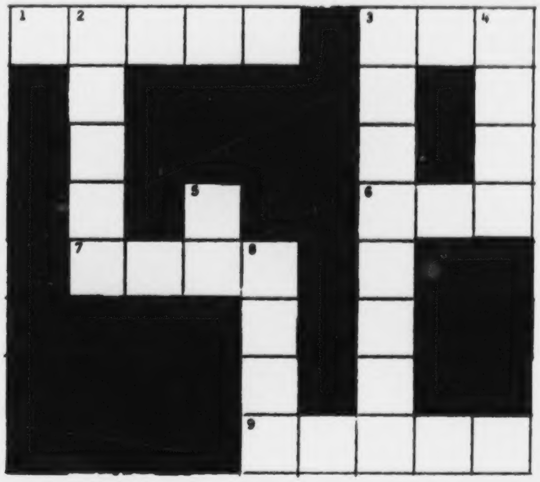


WHOLE NUMBERS

CONCEPT OBJECTIVE: The student will use a calculator to practice whole number operations.

CALCULATOR CROSSTIC

PROCEDURE: The student will need to use a calculator to solve each arithmetic problem. Having found the solution to a problem, invert the calculator and read the word that fits the given clue. Place the word answers in the crossword puzzle.



ACROSS

1	Happiness	$[(106 + 94)10^2 + 7589]2 =$
3	Receding	$911 - (2^2 \cdot 7) =$
6	Jellylike substance	$36064 \div 49 =$
7	Dirt	$20 \cdot 62 + 310 + 1111 \cdot 5 =$
9	A hard outer covering	$3882 + 6589 + 66874 =$

DOWN

2	Doesn't win	$123494 - 9541 - 60446 =$
3	Qualified to participate	$3 \cdot 10^5 \cdot \sqrt{10000} + 7816173 =$
4	Payment due every month	$[(3538 - 333 + 654) \cdot 4] \div 2 =$
5	Greeting	$(139 \cdot 5 - 572) \div 3 =$
8	Not a gain	$5167 - 326 + 1020 - 354 =$

WHOLE NUMBERS

CONCEPT OBJECTIVE: The student will use a calculator to practice multiplication of whole numbers and logical thinking.

HURRAY ARRAY!

PROCEDURE: The student may use the calculator to help fill in the empty boxes in the following square using these clues.

1. Each of the numbers on the outside of the array is the product of all the numbers (missing and visible) in that row or column.
2. Seven of the missing numbers are 2.
3. Five of the missing numbers are multiples of 5, each less than 45.

			6	960
	8			4800
		11		880
3				300
240	1280	1320	3000	

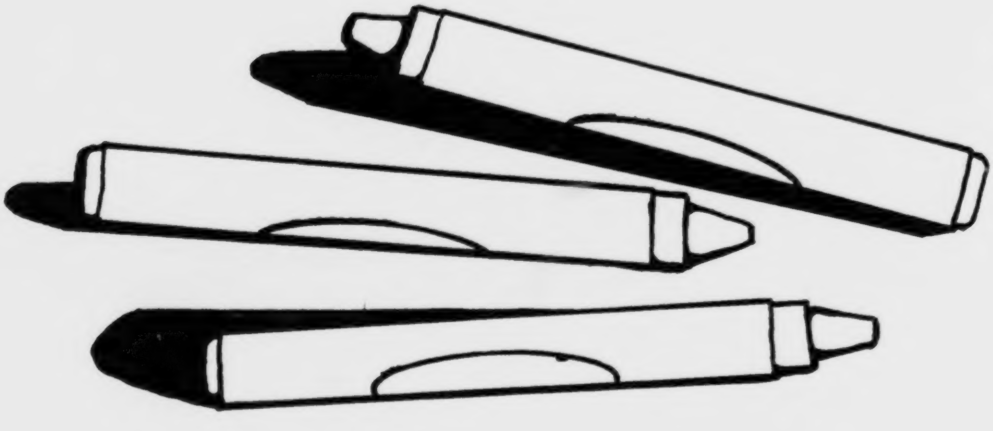
DECIMALS

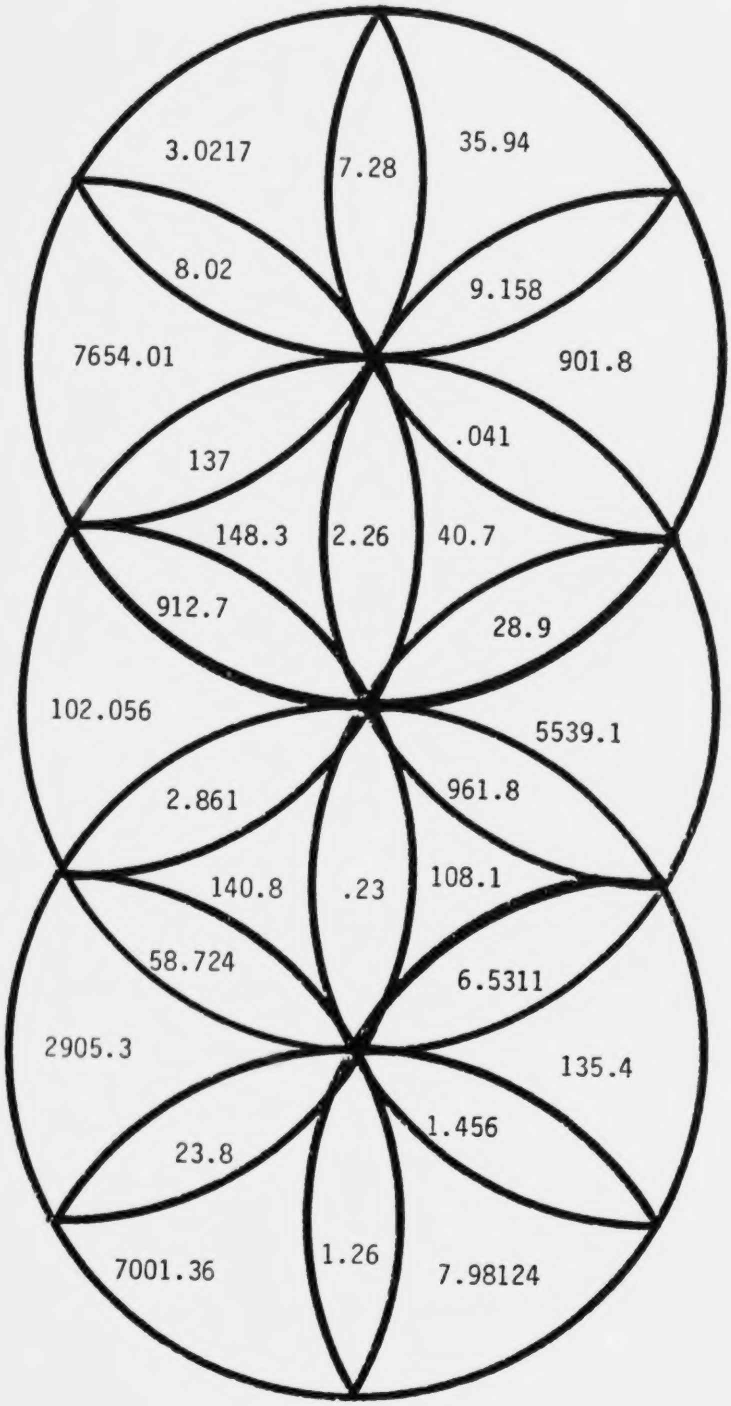
CONCEPT OBJECTIVE: The student will practice identifying decimal place value.

COLOR BY NUMBER

PROCEDURE: Using crayons, colored pencils, or markers, students may color the design as indicated. Students could also be encouraged to determine their own color code and their own designs.

1. Color yellow all sections with a 2 in the tenths place.
2. Color orange all sections with an 8 in the ones place.
3. Color red all sections with a 5 in the hundredths place.
4. Color purple all sections with a 3 in the tens place.
5. Color blue all sections with a 1 in the thousandths place.
6. Color green all sections with a 9 in the hundreds place.
7. Color pink all sections with a 7 in the thousands place.
8. Color brown all sections with a 0 in the ones place.




3.0217
7.28
35.94
8.02
9.158
7654.01
901.8
137
.041
148.3
2.26
40.7
912.7
28.9
102.056
5539.1
2.861
961.8
140.8
.23
108.1
58.724
6.5311
2905.3
135.4
23.8
1.456
7001.36
1.26
7.98124

DECIMALS

CONCEPT OBJECTIVE: The student will apply operations with decimals.

FIRST THANKSGIVING

PROCEDURE: Approximately 120 persons attended the first Thanksgiving feast. Students can determine the cost of the feast using recent prices. With research at the local grocery store, students could use current prices to compute the cost of the feast today.

1. Venison (deer meat)
   If each person ate $\frac{1}{2}$ pound, how many pounds were used?
   ____
   At $1.49 a pound, how much would it cost?______________
2. 100 pounds of duck meat were eaten. At $.98 a pound, it would cost __________.
3. 100 pounds of goose meat were used. If it costs $1.98 a pound, it would cost __________.
4. If 80 of the people liked clams and each ate 6, how many clams were eaten?__________This is __________dozen. At $.95 a dozen they would cost __________.
5. 10 pounds of eel were prepared. At $5.45 a pound, it would cost __________.
6. 10 pounds of haddock, cod, halibut, and flounder were used. At $4.73 a pound, the price would be ___________.
7. If each person ate 4 slices of bread a day and the feast lasted 3 days, how many slices of bread would this be? ___________If there are 20 slices of bread in a loaf, how many loaves would this be?___________If bread costs $1.29 a loaf, the cost would be __________.
8. If a box of Jiffy Corn Bread mix feeds 4 people, how many boxes would be needed for 120 people?_____________ At $.29 a box, it would cost.___________
9. Leeks cost $2.49 a bunch and 5 bunches were used. Cost: __________.

10. Watercress is $1.29 a bunch. Cost of 5 bunches is __________.

11. Spinach is $.89 a bunch. Cost of 5 bunches is __________.

12. Mustard greens are $.59 a bunch. Cost of 5 bunches is __________.

13. If each person ate one ear of corn each day, how many ears were eaten in 3 days? __________ If corn sells for $1.00 for 3 ears, the cost of the corn would be __________.

14. If squash is $.65 a pound, and 50 pounds were used, the cost would be __________.

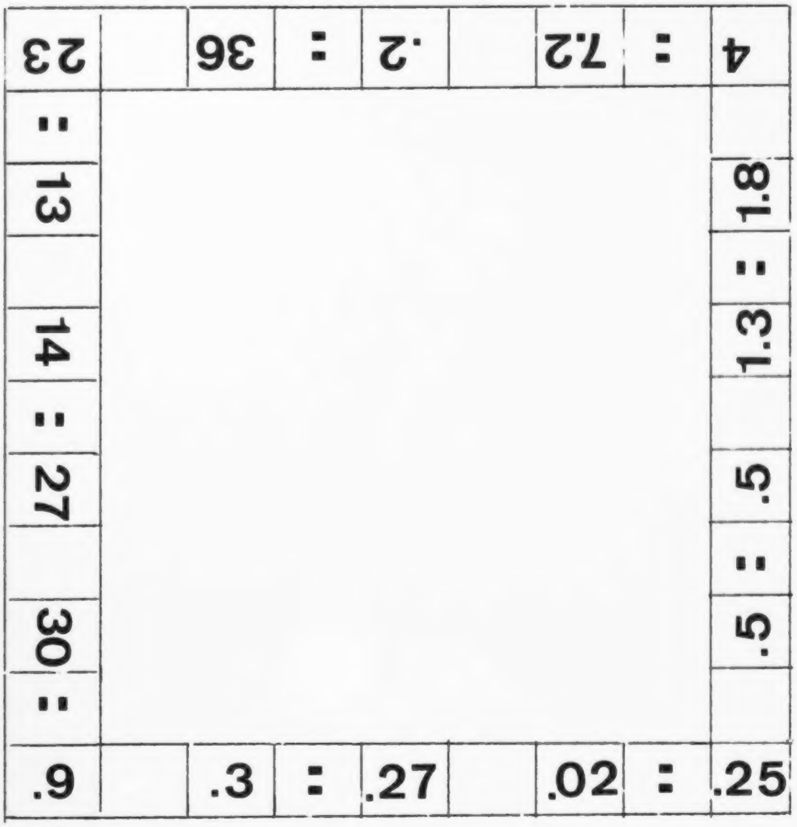
15. The cost of 25 lbs. of wild plums, at $.60 a pound, would be __________.

DECIMALS

CONCEPT OBJECTIVE: The student will practice operations with decimals.

DECIMALS AROUND THE BLOCK

PROCEDURE: Blank spaces will be filled in with the appropriate operation symbol (+, −, x, ÷). The student should begin with .9 and move around the rectangle completing the operation boxes and eventually yielding the final solution of .9 which was the beginning number. Use of a calculator is optional.



START ➤

DECIMALS

CONCEPT OBJECTIVE: The student will practice operations with decimals.

HOW MANY STAMPS?

PROCEDURE: How many ways can a person make a value of $2.43 using only $.05 and $.08 stamps?

CONCEPT OBJECTIVE: The student will use prime factorization to classify terminating and repeating decimals.

PRIME CLUES

PROCEDURE: Given a fraction in lowest terms, the student can find the prime factorization of the denominator. Students could be guided to discover the following conclusions.

If all the prime factors are only 2 or 5, then the fraction will be represented by a terminating decimal. If prime factors other than 2 or 5 are present, the fraction will be represented by a repeating decimal.

DECIMALS

CONCEPT OBJECTIVE: The student will practice changing fractions to decimal numerals.

LONG REPETENDS

PROCEDURE: Students can be challenged to find a fraction which can be changed to a decimal having more than 16 digits in its repetend. An example is:

$$1/19 = \overline{.052631578947368421}$$

The students will find that such fractions will have denominators which are prime numbers. However, not all fractions with prime denominators will have such long repetends.

FRACTIONS

CONCEPT OBJECTIVE: The student will apply set intersection to find the least common denominator.

LEAST COMMON DENOMINATOR

PROCEDURE: The set operation of intersection can be used to determine the least common denominator. Given an addition or subtraction problem of unlike fractions, students may list several multiples in increasing order for each denominator. After the lists have been made, the student should find the intersection of the sets. The smallest number in the intersection will serve as the least common denominator for the given problem.

FRACTIONS

CONCEPT OBJECTIVE: The student will practice adding fractions.

TIC-TAC-TOE

PROCEDURE: Students can find the tic-tac-toe in each table. The tic-tac-toe consists of three fractions whose sum is 1. It may be located vertically, horizontally, or diagonally.

$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{10}$
$\frac{3}{10}$	$\frac{3}{4}$	$\frac{1}{5}$
$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{5}$

$\frac{4}{12}$	$\frac{7}{12}$	$\frac{5}{12}$
$\frac{3}{12}$	$\frac{2}{12}$	$\frac{9}{12}$
$\frac{8}{12}$	$\frac{1}{12}$	$\frac{6}{12}$

$\frac{6}{7}$	$\frac{3}{7}$	$\frac{2}{3}$
$\frac{1}{3}$	$\frac{1}{2}$	$\frac{5}{7}$
$\frac{4}{7}$	$\frac{2}{7}$	$\frac{1}{7}$

$\frac{3}{4}$	$\frac{4}{8}$	$\frac{1}{2}$
$\frac{2}{8}$	$\frac{3}{8}$	$\frac{5}{8}$
$\frac{1}{8}$	$\frac{1}{4}$	$\frac{7}{8}$

$\frac{1}{3}$	$\frac{1}{4}$	$\frac{11}{12}$
$\frac{5}{12}$	$\frac{1}{6}$	$\frac{3}{4}$
$\frac{1}{2}$	$\frac{7}{12}$	$\frac{5}{6}$

FRACTIONS

CONCEPT OBJECTIVE: The student will practice operations with fractions.

FRACTION ROUNDS

PROCEDURE: Each player needs a game sheet and pencil while the leader needs a die or spinner to generate numerals. A student will play four rounds. The player with the largest total at the end of the game is the winner. Beginning with round 1, the leader rolls a die or spins a spinner and calls out the numeral shown. Each individual player decides in which shape to place this numeral. Once the numeral is placed in a shape, the player may not change it. The process continues three more times with the leader rolling the die or spinning the spinner and each player entering the numeral on the game sheet. After the four numerals have been determined for round 1, each player computes the answer to that problem.

Rounds 2-4 are conducted in the same manner. Each player computes the total of the answers from all 4 rounds. The player with the largest total is the winner.

A reproducible game sheet may be found in the appendix.

FRACTIONS

CONCEPT OBJECTIVE: The student will practice operations with fractions.

FOOD FOR THOUGHT

PROCEDURE: The following recipe will make 21 servings in 3 pie shells. Reduce the proportions to serve 7 people from 1 pie shell.

24 ounces of shredded cheddar cheese

3/4 cup of chopped onion

4½ cups diced cooked ham

1 cup flour

1½ teaspoon salt

3/4 teaspoon pepper

3/4 teaspoon caraway seed

3 cups milk

12 eggs

CONCEPT OBJECTIVE: The student will practice operations with fractions.

CLASS PRESIDENT

PROCEDURE: Sherri received 221 votes for class president. If she got 13/20 of all the votes, how many votes were cast in the election?

FRACTIONS

CONCEPT OBJECTIVE: The student will practice operations with fractions.

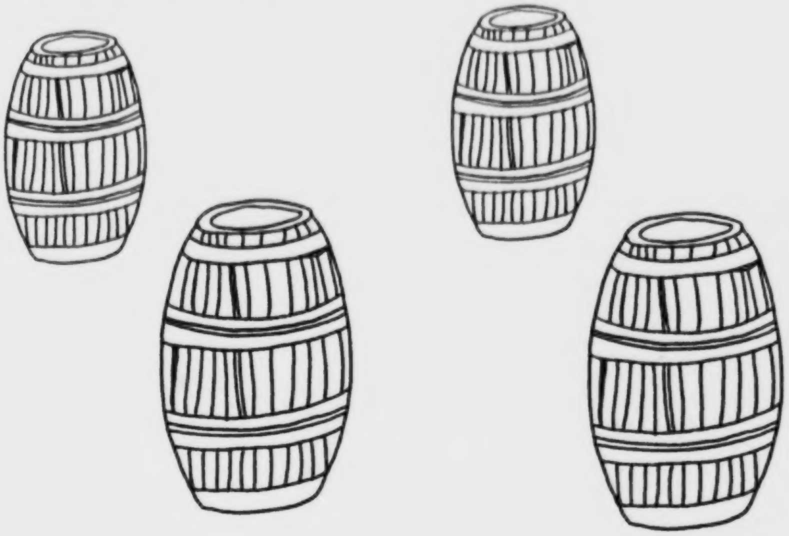
END OF THE ROPE

PROCEDURE: Peter has a rope measuring 38¼ feet long, and he wants to make 6 jump ropes of equal length. How long will each piece be?

CONCEPT OBJECTIVE: The student will practice operations with fractions.

BARRELS OF SAND

PROCEDURE: Sam has received 3/4 ton of sand. If he orders barrels that hold 1/16 ton of sand, how many barrels must he order to store the entire 3/4 ton of sand? Should he decide to order barrels that hold 1/24 ton, how many barrels must he order? Find some possible combinations if he decides to order some barrels of each size.



FRACTIONS

CONCEPT OBJECTIVE: The student will practice operations with fractions.

HAY BALES

PROCEDURE: A storage shed is 45 feet long, 20 feet wide and 16 feet high. Some round bales of hay are $4\frac{1}{2}$ feet in diameter and are $4\frac{1}{2}$ feet tall. Rectangular bales of hay measure 38 inches long, 18 inches wide, and 14 inches high. How many round bales can be placed on the floor of the shed? How many rectangular-shaped bales could be placed on the floor of the shed? If the means to lift the bales is available, how many round bales could be used to fill the shed? How many rectangular-shaped bales would fill the shed?

RATIO AND PROPORTION

CONCEPT OBJECTIVE: The student will practice writing ratios.

STUDENT SURVEY

PROCEDURE: Each student should complete an information survey. The information may be compiled for the entire class in table form. The specific items in each table will be determined by some student answers. After the tables of information have been created, questions of the following type could be discussed.

1. What is the ratio of blonds to the total number of students in our class?
2. What is the ratio of the number of students preferring Coca-Cola to the number of students preferring Pepsi?
3. What is the ratio of the students preferring football to the number preferring basketball?

An example of a student survey may be found in the appendix.

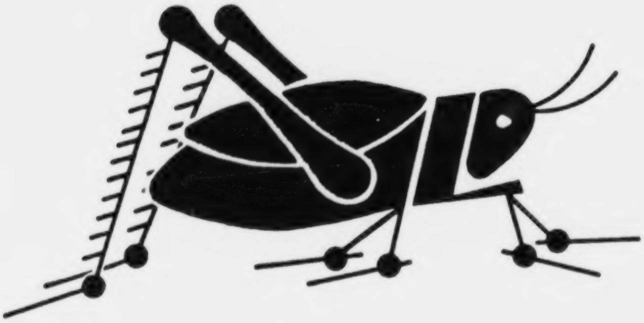
RATIO AND PROPORTION

CONCEPT OBJECTIVE: The student will practice solving a proportion.

CRICKETS

PROCEDURE: There is a relationship between the number of chirps made by a cricket and the temperature where the cricket is located. Students may make the following observations and set up a proportion. On the first day, count the number of chirps made by a cricket and record the temperature where the cricket is located. On another day, count the number of cricket chirps and use a proportion to determine the temperature. Verification can be made with a thermometer.

Students can be encouraged to continue their experiment counting chirps and using a thermometer to determine the temperature. The data can be used to generalize the relationship and construct a formula.



RATIO AND PROPORTION

CONCEPT OBJECTIVE: The student will practice constructing scale drawings.

SCALE DRAWINGS

PROCEDURE: Each student will need $\frac{1}{2}$-inch graph paper, a ruler, scissors and glue or tape. A particular lot for a home has an 80 foot front and is 150 feet deep. On the graph paper, draw the lot to scale using a scale of $\frac{1}{2}$" to 10 feet. Cut out the lot. A 4 foot wide sidewalk is to be constructed on the lot along the front. Draw the sidewalk on the scale figure of the lot and label it.

The dimensions of the house, garden, patio and dog pen to be built on the lot are given below. Draw them on the graph paper to the same scale, cut them out and label each.

House	40' x 50'
Patio	15' x 20'
Garden	10' x 15'
Dog Pen	10' x 10'

Glue or tape the four features to the scaled lot. Identify the area and perimeter of the lot and each of the four features.

PERCENT

CONCEPT OBJECTIVE: The student will change fractions to percents.

TAKE TIME FOR PERCENT

PROCEDURE: Students can select their own radio or television station and a given time period. They can design their own methods of recording the amount of time spent for different types of broadcasting, i.e., commercials, news, music, announcements. From each set of data, the percent of total time spent for each type of programming can be determined.

PERCENT

CONCEPT OBJECTIVE: The student will practice finding percents.

SURVEY PERCENTS

PROCEDURE: Information gathered from a student survey may be used in a percent activity. An example of a student survey may be found in the appendix.

Each student should complete a survey form and information for the entire class compiled in table form. Specific items in each table will be determined by some student answers. After the tables of information have been created, questions such as the following may be answered.

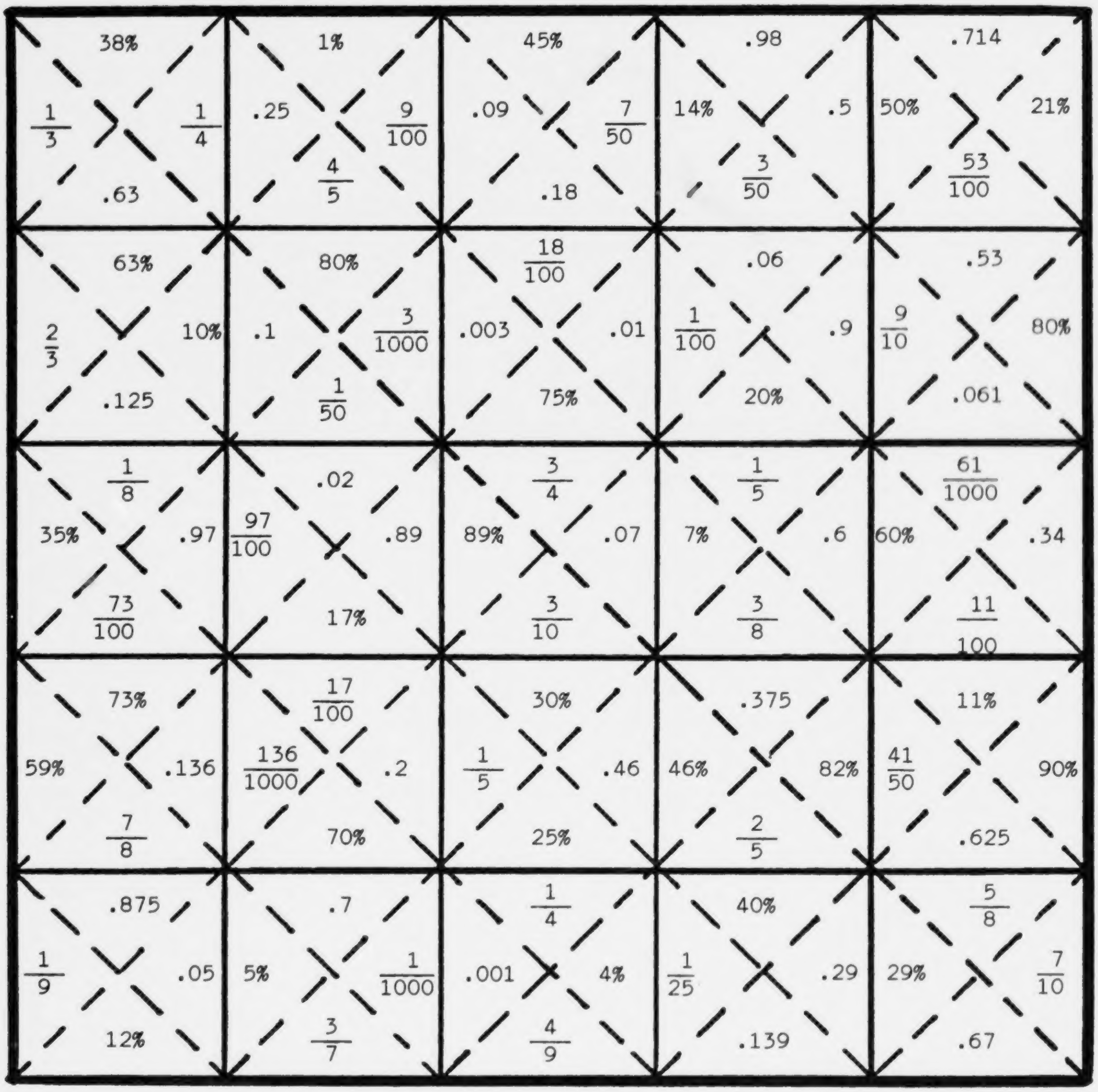
1. What percent of our class has no brothers or sisters?
2. What percent of our class has 3 or more pets?
3. What percent of our class was born in October?

CONCEPT OBJECTIVE: The student will change fractions, decimals, and percents into equivalent expressions.

PERCENTS, DECIMALS, AND FRACTIONS

PROCEDURE: After duplicating the puzzle, cut it into the 25 small squares. The students can reassemble the original square by matching equivalent fractions, decimals, and percents. The puzzle will be easier for students to put together if the wide outside border is not trimmed.

PERCENTS, DECIMALS, AND FRACTIONS



38% $\frac{1}{3}$ $\frac{1}{4}$ .63	1% .25 $\frac{9}{100}$ $\frac{4}{5}$	45% .09 $\frac{7}{50}$ .18	.98 14% .5 $\frac{3}{50}$	.714 50% 21% $\frac{53}{100}$
63% $\frac{2}{3}$ 10% .125	80% .1 $\frac{3}{1000}$ $\frac{1}{50}$	$\frac{18}{100}$ .003 .01 75%	.06 $\frac{1}{100}$ .9 20%	.53 $\frac{9}{10}$ 80% .061
$\frac{1}{8}$ 35% .97 $\frac{73}{100}$	.02 $\frac{97}{100}$ .89 17%	$\frac{3}{4}$ 89% .07 $\frac{3}{10}$	$\frac{1}{5}$ 7% .6 $\frac{3}{8}$	$\frac{61}{1000}$ 60% .34 $\frac{11}{100}$
73% 59% .136 $\frac{7}{8}$	$\frac{17}{100}$ $\frac{136}{1000}$ .2 70%	30% $\frac{1}{5}$ .46 25%	.375 46% 82% $\frac{2}{5}$	11% $\frac{41}{50}$ 90% .625
.875 $\frac{1}{9}$ .05 12%	.7 5% $\frac{1}{1000}$ $\frac{3}{7}$	$\frac{1}{4}$ .001 4% $\frac{4}{9}$	40% $\frac{1}{25}$ .29 .139	$\frac{5}{8}$ 29% $\frac{7}{10}$ .67

MEASUREMENT

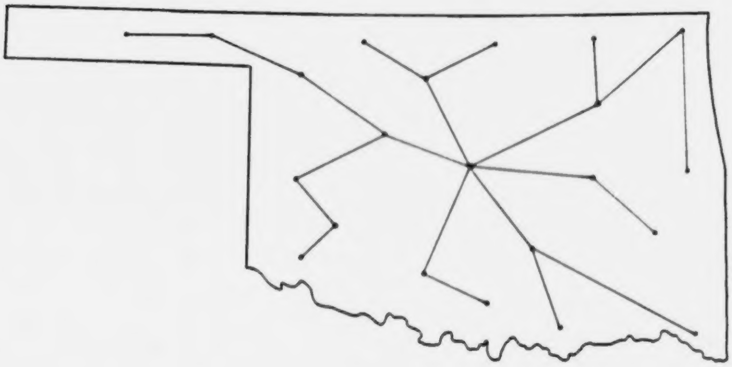
CONCEPT OBJECTIVE: The student will practice map-reading skills.

MAP-READING

PROCEDURE: Many mathematical skills and concepts can be reinforced while giving students practice in reading maps. With a specific map identified for use, have students answer a set of questions written to practice some of the following skills.

1. Identify symbols used on the map and locate county seats, state/national parks, and tourist information centers.
2. Read the mileage scale.
3. Find the distance between 2 locations.
4. Read elevations.
5. Give and follow directions to travel from one location to another.
6. Determine population and order towns according to population size.
7. Use the map coordinate system to locate towns.

Students could also compose individual sets of questions to be shared among other students.



MEASUREMENT

CONCEPT OBJECTIVE: The student will practice constructing line segments and angles.

FLY A KITE

PROCEDURE: Each student will need a standard ruler, protractor, and and a piece of paper.

1. Locate Point P 2 inches above the bottom edge of the paper and 2 inches from the right hand edge of the paper.
2. Draw $\overline{PM}$ 1 inch long. (M is to the right of P and $\overline{PM}$ is parallel to the bottom edge of the paper.)
3. $\overline{PQ}$ is $1\frac{1}{2}$ inches long and $\measuredangle MPQ = 155^\circ$. (Q is above $\overline{PM}$.)
4. $\overline{QR}$ is $1\frac{1}{2}$ inches long and $\measuredangle PQR = 140^\circ$. (R is above Q.)
5. $\overline{RS}$ is 1 inch long and $\measuredangle QRS = 140^\circ$. (S and P are on the same side of $\overline{RQ}$.)
6. $\overline{ST}$ is $4\frac{1}{4}$ inches long and $\measuredangle RST = 165^\circ$. (T and Q lie on opposite sides of $\overline{RS}$.)
7. $\overline{TU}$ is 2 inches long and $\measuredangle STU = 100^\circ$. (U and M lie on opposite sides of $\overline{RS}$.)
8. $\overline{UV}$ is $2\frac{1}{4}$ inches long and $\measuredangle TUV = 95^\circ$. (V and S lie on the same side of $\overline{UT}$.)
9. Connect V to S.
10. $\overline{QN}$ and $\overline{QO}$ are $\frac{1}{2}$ inch long and $\measuredangle RQN$ and $\measuredangle RQO = 90^\circ$.
11. $\overline{QL}$ and $\overline{QF}$ are $\frac{1}{2}$ inch long and $\measuredangle PQL$ and $\measuredangle PQF = 90^\circ$. (L is on the same side of $\overline{RQ}$ as N.)
12. Connect L to N. Connect F to O.
13. $\overline{RD}$ and $\overline{RE}$ are $\frac{1}{2}$ inch long and $\measuredangle SRD$ and $\measuredangle SRE = 90^\circ$.
14. $\overline{RB}$ and $\overline{RC}$ are $\frac{1}{2}$ inch long and $\measuredangle QRB$ and $\measuredangle QRC = 90^\circ$. (B is on the same side of $\overline{RS}$ as D.)
15. Connect B to D. Connect C to E.

MEASUREMENT

CONCEPT OBJECTIVE: The student will practice finding perimeter and area.

AN IDEAL COURT

PROCEDURE: Students can measure the length and width of the school basketball court. They can determine the perimeter and area of the court. According to the National Federation Basketball Rulebook, the dimensions for an ideal basketball court for high school play is 50 ft. by 84 ft. Comparisons between the school basketball court and the ideal court can be made.

Students can find the length of the radius of each circle at mid-court. On the ideal court, the circles would have radii of 2 ft. and 6 ft. Comparisons can also be made between the length of the diagonals on the school court and the ideal basketball court.

CONCEPT OBJECTIVE: The student will apply the concept of area in a problem solving situation.

CUT A RUG

PROCEDURE: Given a rug which is 12 ft. by 12 ft., ask students to show it is possible to cut the rug into exactly 2 pieces and place them together so they will fit into a 9 ft. by 16 ft. room.

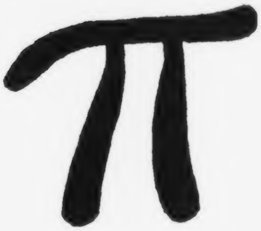
MEASUREMENT

CONCEPT OBJECTIVE: The student will determine an approximate numerical value for pi.

PI

PROCEDURE: Students can perform the following activity to derive their own approximate value for pi, the ratio of the circumference to the diameter of a circle. A length of string should be laid flat. Select a wheel, coin or other circular object. Beginning carefully at one end of the string, roll the object one complete revolution along the string. Cut the string at the point where one revolution is completed.

The piece of string represents the circumference of the circle. Use the string to determine how many diameter lengths are represented in the circumference. Students should determine that there are a little more than three diameter lengths.

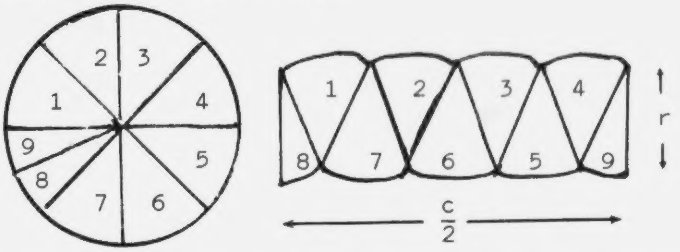


MEASUREMENT

CONCEPT OBJECTIVE: The student will be guided through the development of the formula for the area of a circle.

AREA OF A CIRCLE

PROCEDURE: To develop the formula for the area of a circle, use the rectilinear rearrangement of equal sectors of a circle. The circle can be divided into 9 sectors as shown. Placing the sectors in a rectangular arrangement gives a width of r, the radius of the circle, and a length of C/2, where C is the circumference of the circle.



Using the formula for the area of a rectangle and substitution, the formula for the area of a circle can be derived.

$$A = l \times w$$

$$A = \frac{c}{2} \times r$$

$$C = 2\pi r$$

$$A = \frac{2\pi r}{2} \times r$$

$$A = \pi r^2$$

MEASUREMENT

CONCEPT OBJECTIVE: The student will practice finding volume.

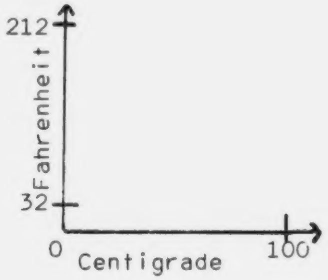
PERSONAL SPACE

PROCEDURE: Students can find the volume of the classroom. The volume of large pieces of furniture may also be determined. Subtracting the volumes of the furniture from the volume of the classroom leaves the space in the room available for students and teacher. A student can calculate how much personal space he or she is allowed by dividing the available volume by the number of persons in the classroom. Comparisons can be made among the personal space allotments in other classrooms and in rooms at home.

CONCEPT OBJECTIVE: The student will graphically represent the relationship between Fahrenheit and Centigrade measures of temperature.

TEMPERATURE SCALE

PROCEDURE: Students can draw a set of axes and identify the vertical axis as Fahrenheit and the horizontal axis as Centigrade. Since the freezing temperature of water is 0° C and 32°F, the point (0,32) should be placed on the graph. The boiling temperature of water is 100°C and 212°F. This point can be graphed. Since the relationship between the temperature scales is a linear one, the student may connect the 2 points. Given a value in one scale, the graph may be used to estimate the temperature in the other scale. Students may also be encouraged to generalize the relationship and construct a formula for converting between temperature scales.



GRAPHING

CONCEPT OBJECTIVE: The student will construct bar, broken-line, and picture graphs.

INFORMATION AT A GLANCE

PROCEDURE: Information gathered from a student survey may be used in a graphing activity. An example of a student survey may be found in the appendix. Each student should complete a survey and the information for the entire class compiled. Using these class information tables, bar, broken-line, or picture graphs may be constructed.

CONCEPT OBJECTIVE: The student will plot ordered pairs in the first quadrant.

LITTLE DIPPER

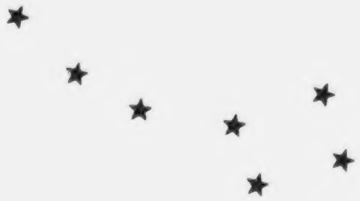
PROCEDURE: Students may plot and connect the following points in order.

(6½, 17½) (12½, 20) (18, 24½) (21½, 31½) (27½, 31½)

(28, 39) (22, 40)

Stars can be placed at the location of the named points to give the appearance of a constellation. The figure is a representation of URSA MINOR (Little Dipper).

Other constellations can be graphed using ordered pairs on a set of coordinate axes to locate the stars.



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GRAPHING

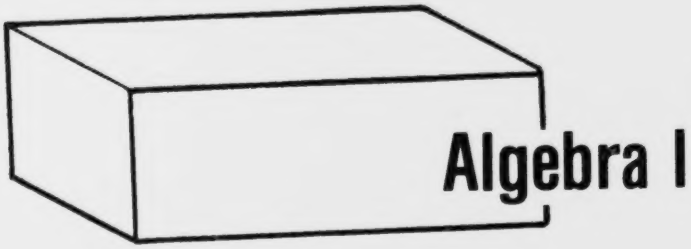
CONCEPT OBJECTIVE: The student will practice assigning ordered pairs to points in the coordinate plane.

CREATIVE DESIGN

PROCEDURE: Students can practice coordinate graphing by drawing an original design on a set of axes, assigning ordered pairs and listing points in order of connection to recreate the picture. Exchanging directions with another student could serve as a check to see if someone else could reproduce the same design.

42

ALGEBRA ONE
VARIABLES, EQUATIONS, APPLICATIONS
THINK, THINK, THINK



ALGEBRA I

Algebra I provides the student with the beginning skills to generalize arithmetic operations. The concept of a variable and its use in equations to solve problems is very important. The student experiences a broadening and extension of the properties of real numbers by working with algebraic expressions, polynomials, inequalities and various functions. Word problems which require algebraic solutions help students develop problem solving skills. Real-life applications of algebraic concepts help acquaint students with the usefulness of mathematics.

TOPIC	PAGE
REAL NUMBERS	39
EQUATIONS	48
POLYNOMIALS	54
GRAPHING	60
SYSTEMS OF EQUATIONS	62

The contents of this chapter are organized by four strands coded with symbols as follows:

✏ - Activity

🎲 - Game

🌎 - Application

🔍 - Problem-solving Experience

Many activities are coded under two or more strands.

REAL NUMBERS

CONCEPT OBJECTIVE: The student will practice adding integers.

INTEGER MAGIC SQUARES

PROCEDURE: Students can complete each magic square. They should verify that the sum of each row, column, and diagonal is the same number and state that sum for each magic square.

3		1
	0	
-1		

6	-1	4
		0

	-5	
-8	-1	-6

		-8
-14		-6
		-16

REAL NUMBERS

CONCEPT OBJECTIVE: The student will practice ordering integers.

WAR

PROCEDURE: Each pair of players needs a deck of Integer cards. After the cards have been shuffled, the dealer deals all the cards face down to both players. Each player leaves his/her cards in a face-down stack, turns over the top card, and places it in the middle of the table. The player having the card with the largest integer collects both cards played. If the cards have the same integer, each player turns over another card and the player with the largest integer claims all of the cards. After all cards have been played once, each player turns over the cards that have been collected and uses them as a new face-down stack. The process continues until one player has collected all the cards or until time is called. At the end of the time period, the player having the most cards is the winner.

Integer cards are composed of two each of the following for a deck of 42 cards: 0, 1, -1, 2, -2, 3, -3, . . . , 10 -10.

REAL NUMBERS

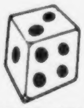
CONCEPT OBJECTIVE: The student will develop an understanding of multiplication of positive and negative numbers.

POSITIVE AND NEGATIVE NUMBERS

PROCEDURE: The rules for the sign of a product can be illustrated in a simple experiment. A movie film can show a person putting eight beads on a string per minute. Running the film forward for five minutes will show forty beads placed on a string. This illustrates the number sentence: (+5)(+8) = +40. Running the film backward for five minutes will show forty beads being removed from the string, illustrating the number sentence: (-5)(+8) = -40.

Another movie film can be made showing a person removing eight beads per minute from a long string. Running the film forward for five minutes will show forty beads being removed from the string. This action illustrates the number sentence: (+5)(-8) = -40. Running the film backward for five minutes will show forty beads being placed on the string, illustrating the number sentence: (-5)(-8) = +40.

CONCEPT OBJECTIVE: The student will practice operations with integers.



INTEGER INVITATIONAL

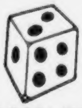
PROCEDURE: Each pair of players needs a score card, a pencil, and one pair of dice of different colors. Players should agree among themselves to designate one colored die as positive and the other as negative. Each player rolls the positive die and the player with the highest number begins golfing. The first golfer rolls both dice at the same time until the rule stated for Hole 1 is met. The number of times the dice are rolled to meet the rule is the player's score on Hole 1.

The other player rolls to determine his/her score for Hole 1. Play continues until all 9 holes are completed. The scores are added and the player with the lowest total is the winner.

A reproducible score card may be found in the appendix.

REAL NUMBERS

CONCEPT OBJECTIVE: The student will practice ordering rational numbers.



PUT-IN-ORDER

PROCEDURE: Each group of 2-6 students will need a deck of Put-in-Order cards. After shuffling, the dealer places five cards in a horizontal row, face up in front of each player. A player may not move or rearrange the cards. The remaining cards are placed in a stack face down in the middle of the table. The player to the left of the dealer begins the play. The goal for each player is to get five cards in order from smallest to largest. None of the fractions may be equivalent. In turn, a player selects one of the fraction cards to discard, and places the top card from the deck in its place. The card that is discarded is placed on the bottom of the deck. At no time may a player change the position of the cards on the table. The game continues as each player selects a fraction to discard and replaces it with the top card from the deck. The first player to have five cards in order wins.

Put-in-Order cards are composed of two each of the following:

$\frac{0}{2}, \frac{1}{2}, \frac{0}{3}, \frac{1}{3}, \frac{2}{3}, \frac{0}{6}, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{-1}{2}, \frac{-1}{3}$

$\frac{-2}{3}, \frac{-1}{6}, \frac{-2}{6}, \frac{-3}{6}, \frac{-4}{6}, \frac{-5}{6}$, and one each of these

$\frac{2}{2}, \frac{3}{3}, \frac{6}{6}, \frac{-2}{2}, \frac{-3}{3}, \frac{-6}{6}$ to make a deck of 44 cards.

REAL NUMBERS

CONCEPT OBJECTIVE: The student will demonstrate the reasons why division by zero is undefined.

DIVISION BY ZERO

PROCEDURE: Division by zero is undefined. However, the reasons behind the statement are often not explained. Students can be guided to discover the reasons by considering the following procedures.

First, look at the case of a nonzero number divided by *0*. As an example, consider *4÷0*. By definition of division, the solution would be x, such that x times *0* is equal to *4*. There is no number, x, for which that statement is true. Thus, *4÷0* is undefined because there is no solution.

$$\begin{array}{r} x \\ 0\overline{)4} \end{array}$$

The second case is that of zero divided by zero, *0÷0*. By definition of division, the solution would be x, such that x times *0* is equal to *0*. Any number, x, will make the statement true. Thus, *0÷0* is undefined because there are an infinite number of solutions.

$$\begin{array}{r} x \\ 0\overline{)0} \end{array}$$

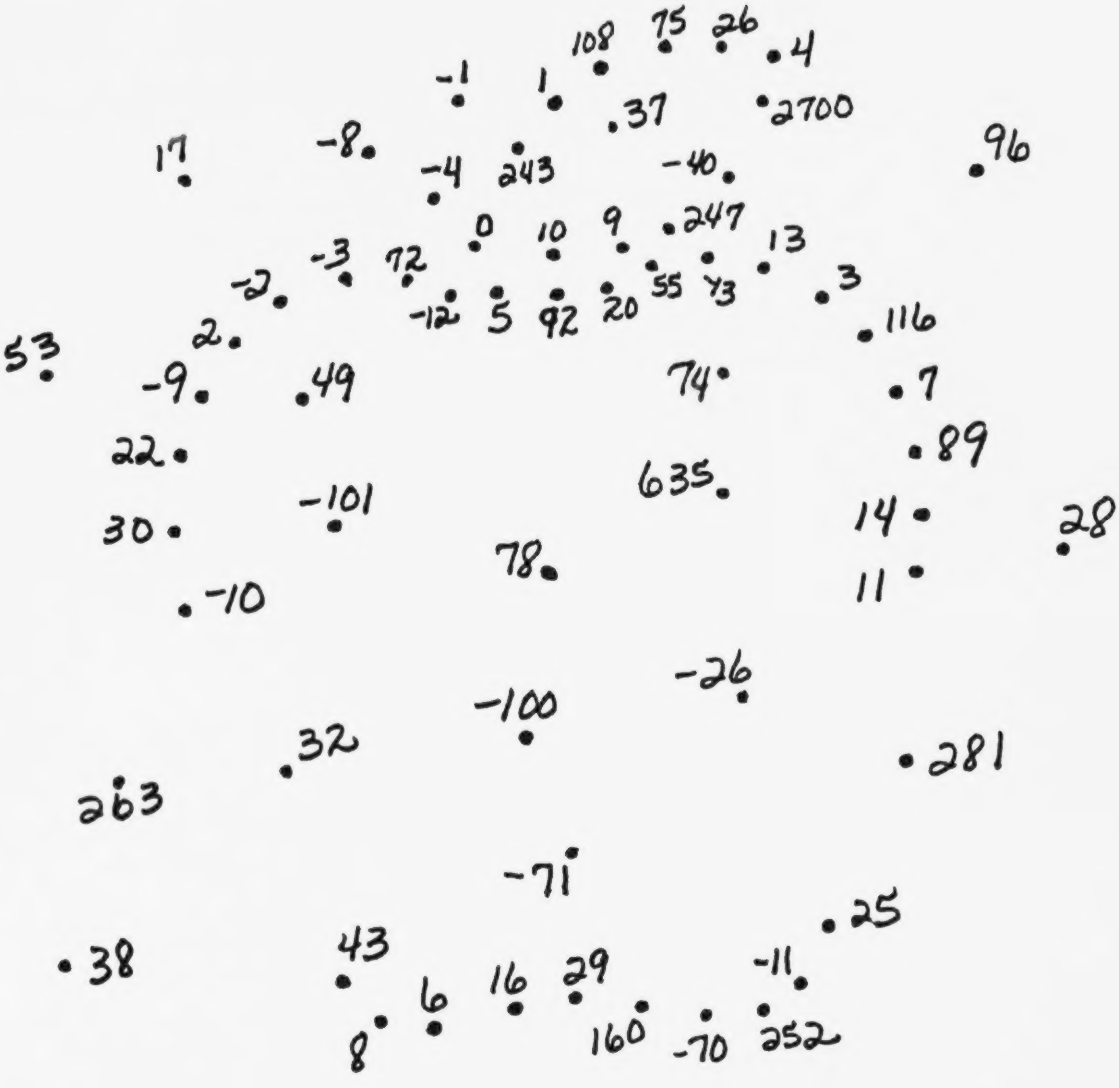
REAL NUMBERS

CONCEPT OBJECTIVE: The student will practice operations with real numbers.

SCHOOL DAZE PUZZLE

PROCEDURE: Students can discover the picture hidden in the dots by first simplifying the expressions below. To graph the picture, begin by locating the dot next to the answer of the first problem. Connect this dot with the dot for the answer to the second problem. Continuing this process, the student should create a recognizable figure. Not all of the dots and numbers will be used.

1. $20 + 20 \div 5 + 5$
2. $24 - 4 \div \frac{1}{2}$
3. $6 \times 5 - 8 \times 3$
4. $25 \div (6-1) + 3$
5. $25 + 3(10 - 4)$
6. $31 + (-62) + 21$
7. $(-40)\cdot(6)\cdot\frac{(-1)}{8}$
8. $7 - (-15)$
9. $-15 + (-24 + 30)$
10. $-2[4 - (3 + 2)]$
11. $-1[2 + 2(-8)] \div (-14 + 7)$
12. $-|4 - 7|$
13. $36 + 4[1 + (12 - 8) \times 2]$
14. $|4 - 1| - 3|2 - 7|$
15. $|-10| - |-5|$
16. $|12| - |-8| - |4|$
17. $-(|-9| - |5|)$
18. $(-2)^3(1)^2$
19. $-(7 - 2 \times 3)^2$
20. 3^5
21. $(7 - 2 \times 3)^3$
22. $12(5^2 - 4^2)$
23. $(7^2 - 34)(1^2 + 2^2)$
24. $7 \times 3 - 2^2 + 3^2$
25. $(2)^5(\frac{1}{2})^3$
26. $10^2 \times 3^3$
27. $5(-2)^3$
28. $2.47(100)$
29. $.27 \div (.03)$
30. 25% of 40
31. 50% of 184
32. 10% of 200
33. $\sqrt{3025}$
34. $-\sqrt{169}$
35. $\sqrt{36} + \sqrt{49}$
36. $-\sqrt[3]{-27}$
37. $4[3(3) + 4(5)]$
38. $[5(5) + 4(6)] \div 7$
39. $11^2 - (2)(4)^2$
40. $\frac{8^2 - 6^2}{8 - 6}$
41. $(4^3 \div 2^4) + 7$
42. $7(2^2) - 15 \div 5$
43. $-|-5 + 18| + |-2|$
44. $-1(-252)$
45. $-(63 + 4) + 4 + (-7)$
46. $-8[-12 + (-8)]$


108
75
26
4
-1
1
37
2700
17
-8
-4
243
-40
96
0
10
9
247
13
-3
72
-2
55
43
3
-12
5
92
20
116
2
53
-9
49
74
7
22
89
635
-101
14
28
30
78
11
-10
-26
-100
32
281
263
-71
25
38
43
6
16
29
-11
8
160
-70
252

REAL NUMBERS

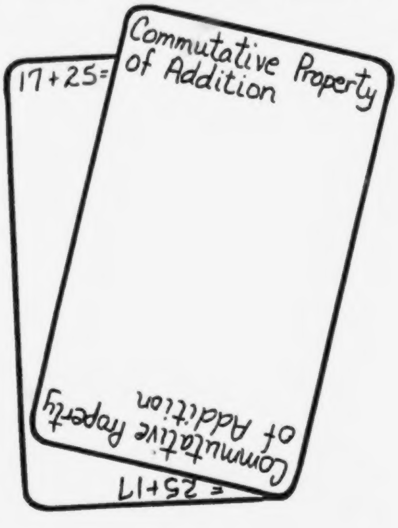
CONCEPT OBJECTIVE: The student will practice identifying properties of real numbers.

PAIRING PROPERTIES

PROCEDURE: Each group of 3-5 students needs a deck of Property cards. After the cards have been shuffled, the dealer deals out all of the cards to the players. Each player views his/her hand and removes any pairs of cards. A pair consists of a property card and an example card demonstrating that property. The object of the game is to obtain as many pairs as possible.

The player to the left of the dealer begins play by drawing a card from the hand of the person on his/her left. Proceeding clockwise around the group, each player draws one card from the player on the left. Players should continue to place matching pairs from their hands in front of them. The first player to match all of the cards in his/her hand receives 1 point for each card remaining in the hands of all other players. Each player receives 5 points for each pair of cards he/she matched. The winner of the game is the first player to accumulate 50 points.

Property cards are created in pairs. For each pair, one card names a property of real numbers, such as the Commutative Property of Addition, and the other card gives an example of that property using real numbers, such as 17 + 25 = 25 + 17. A deck of 15-20 pairs would be adequate.


17+25=
Commutative Property of Addition
= 25+17
Commutative Property of Addition

REAL NUMBERS

CONCEPT OBJECTIVE: The student will practice evaluating algebraic expressions.

EVALUATING MAGIC

PROCEDURE: Given the following magic square, students must first evaluate each algebraic expression for the given values. The value of the expression should be placed in each square. If the work is accurate, the sum of every row, column, and diagonal should be the same.

Use $r = 1$, $s = 2$, $t = 3$, $w = 10$, $x = 1$, and $y = 0$ to evaluate each expression.

$\frac{w-rs}{n-x}$	$\frac{y-(r+s)}{w-3n}$	$\frac{s(w-n)}{xw-(r+s)}$
$\frac{sn-w}{n+s-r}$	$\frac{wx-s}{3n-x}$	$\frac{rn-y}{x(n-s)}$
$xy(n+s)$	$\frac{rsnwx}{w+rs}$	$\frac{sn+wx}{s-w}$

REAL NUMBERS

CONCEPT OBJECTIVE: The student will find the absolute value of rational numbers.

ABSOLUTE VALUE

PROCEDURE: Interpreting absolute value as distance may assist students' understanding. Colored chalk may also be used to clearly represent absolute value expressions graphically. For simple equations involving the absolute value of a variable, such as $|x| = 4$, the number line can be divided into three sections. One color of chalk would indicate that $x = 4$ or $x = -4$. Another color would be used to indicate the area where $|x|$ would be less than 4 and a third color used to show the location where $|x|$ would be greater than 4.

EQUATIONS

CONCEPT OBJECTIVE: The student will practice solving problems using equations.

102 COOKIES

PROCEDURE: Julie ate a total of 102 cookies over a period of 6 days. Each day she ate 4 more cookies than on the previous day. How many cookies did she eat on the first day?

EQUATIONS

CONCEPT OBJECTIVE: The student will solve linear equations in one variable.

CHOCOLATE ALGEBRA PIE

PROCEDURE: Students can decode the following recipe by solving each equation. Some may even want to check their work by tasting.

1. Cream x cups of butter with y cups of sugar, where x and y are respective solutions for the equations

   $6x - (4x + 9) = -8$

   $5y - (3 - 2y) = 3 - y$

2. Stir in z squares of cooled melted unsweetened chocolate, where z is the solution to the equation

   $-3z + 4(3z - 1) = 16 - z$

3. Add m eggs, one at a time, beating at high speed 5 minutes after each addition, where m is the solution for the equation

   $7 - 4m = 12m - 25$

4. Fold in n cups of whipped topping (thawed), where n is the solution for the equation

   $2n - (2 + n) = 0$

5. Pour into a cooled baked 9-inch pie shell. Chill until firm, about 2 hours, or freeze.

EQUATIONS

CONCEPT OBJECTIVE: The student will practice solving problems using equations.

TAKE HOME PAY

PROCEDURE: James works for \$6.25 per hour. Of the total earned, 25% is deducted for income tax, social security and insurance. How many hours must he work in a week to take home \$250?

CONCEPT OBJECTIVE: The student will practice logical thinking and solving linear equations.

CODED EQUATIONS

PROCEDURE: The letters in the following series of equations represent the digits 0-9. Determine which letter represents each digit.

W plus G equals G

J plus N equals L

M times R equals (10•G) + M

N plus B equals (10•G) + M

B times L equals (10•S) + J

S minus J equals M

R plus M equals L

C times C equals (10•G) + N

G times R equals R

EQUATIONS

CONCEPT OBJECTIVE: The student will practice solving problems using equations.

OLD TEX'S GOLD

PROCEDURE: Old Tex told his grandson, Li'l Tex, about some gold buried under a large flat rock 1/3 of the way from the large gate to the lake on his property. The old line shack is 3 km west of the large gate and the windmill is 1 km west of the line shack. The lake is west of the line shack. He gave the following instructions:

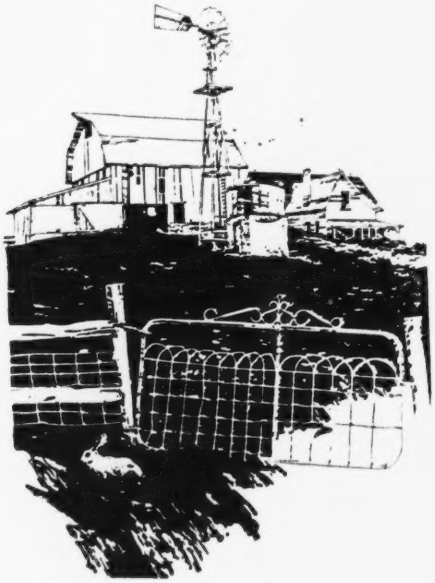
1. Start at the large gate and go west 3/4 the distance to the lake.

2. Then travel east 2/3 the distance back to the gate.

3. Turn and go west 1/4 the distance from the gate to the windmill.

4. There he will find the gold.

Find the distance from the windmill to the lake and where the gold is buried.

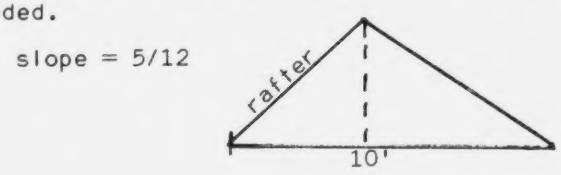


EQUATIONS

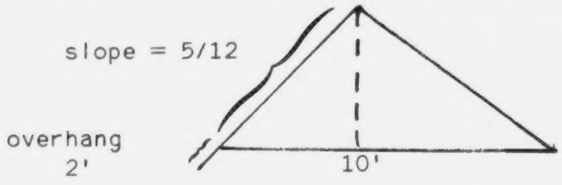
CONCEPT OBJECTIVE: The student will practice using the concept of slope to solve problems.

SLOPING RAFTERS

PROCEDURE: In building construction, it is necessary to determine the length of the roof rafters before they are cut. In a particular building, the length from the middle of the ceiling joist to the outside of the building is 10 feet. The slope of the rafter must be 5/12. Determine the length needed.



Rafters are cut to allow an additional 2 foot overhang. Determine the total length of the rafter to be cut.



EQUATIONS

CONCEPT OBJECTIVE: The student will practice solving problems using equations.

DRIBBLES

PROCEDURE: Students can measure the school basketball court to determine the length and width. The National Federation Basketball Rule Book cites the dimensions of an ideal court for high school play. Using the following information, students can determine those ideal dimensions.

1. The semiperimeter is 134 feet.
2. The sum of one and the square of a number, x, represents the width.
3. The length is twelve times the number, x.

Students can compare their school court dimensions, perimeter, and area with those of the ideal court.

CONCEPT OBJECTIVE: The student will demonstrate the concepts present in rate-time-distance problems.

TRAVELING

PROCEDURE: Simulation can be used to provide students with a concrete representation of two vehicles traveling at different rates for the same length of time. Two students can be asked to represent the two vehicles, moving toward each other or away, as stated in a particular problem. The students should start together, move at different speeds, and stop at the same time. Thus, even though the time traveled is equal, the rate and distance covered are not the same.

EQUATIONS

CONCEPT OBJECTIVE: The student will practice solving fractional equations.

BANANA BREAD

PROCEDURE: *Algebra II, page 93.

CONCEPT OBJECTIVE: The student will practice logical thinking.

3 = -3

PROCEDURE: *Algebra II, page 95.

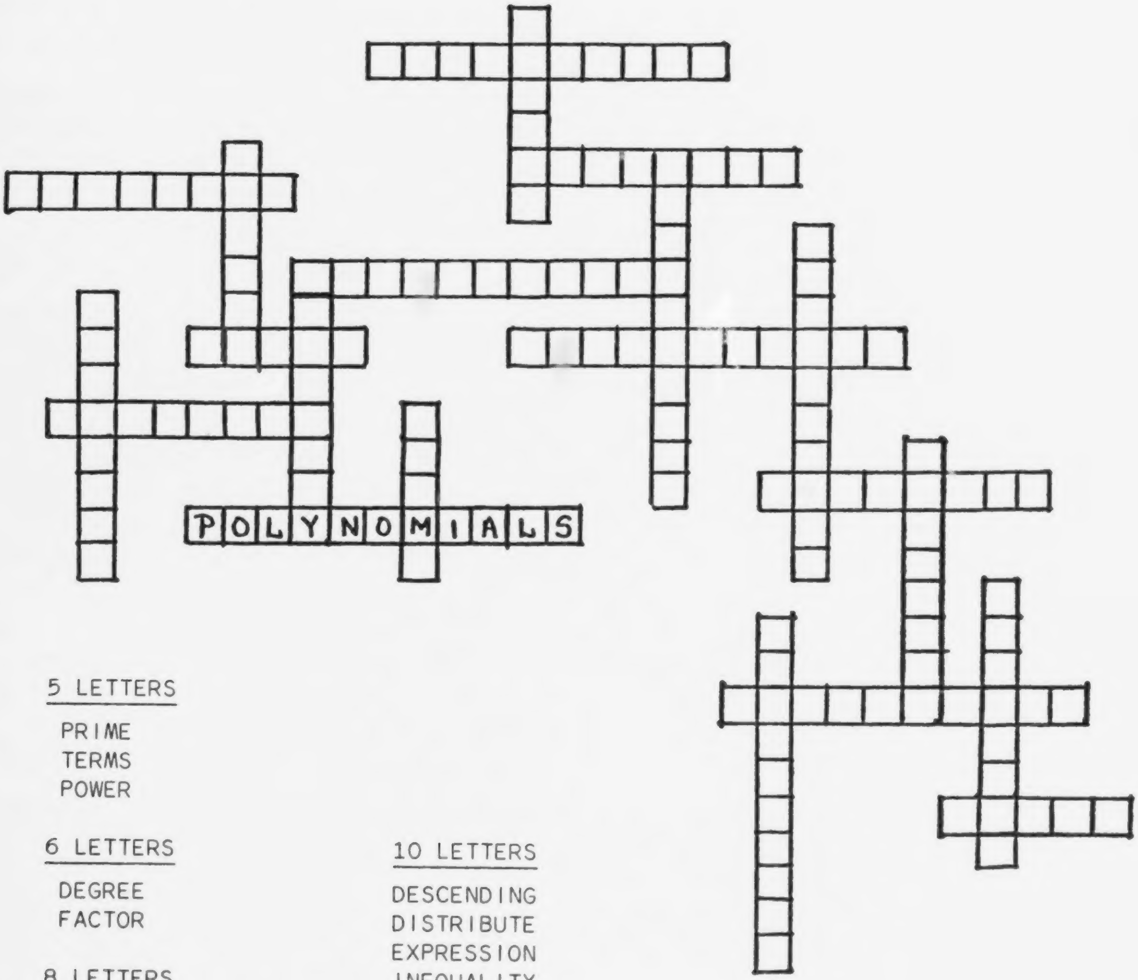
POLYNOMIALS

CONCEPT OBJECTIVE: The student will become familiar with algebraic vocabulary.

ALGEBRAIC CRISS-CROSS

PROCEDURE: Students can complete the puzzle by determining the location of each word in the list and filling in the squares. Each word in the list will be used. Similar puzzles may be created for any topic.

ALGEBRAIC CRISS-CROSS



5 LETTERS

PRIME
TERMS
POWER

6 LETTERS

DEGREE
FACTOR

8 LETTERS

BINOMIAL
MONOMIAL
ADDITION
DIVISION
EQUATION
EXPONENT
MULTIPLY
SIMPLIFY

10 LETTERS

DESCENDING
DISTRIBUTE
EXPRESSION
INEQUALITY
TRINOMIALS

11 LETTERS

DENOMINATOR
✓POLYNOMIALS
SUBTRACTION

POLYNOMIALS

CONCEPT OBJECTIVE: The student will practice operations with polynomials.

TRY A DIGIT

PROCEDURE: Use each of the digits, 1-9, only one time to correctly complete the following set of polynomial problems.

1.
$$\begin{array}{l} \square M + 2N \\ \underline{\square M + \square N} \\ 12M + \square N \end{array}$$

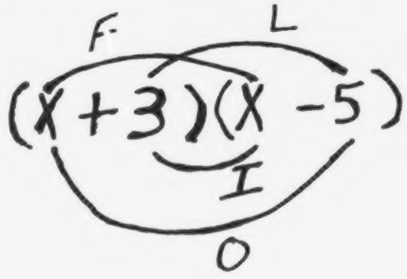
2. $(\square X + 4) - (5X - \square) = \square X + 13$

3. $(2A + \square)(A + \square) = 2A^2 + 5A + 2$

CONCEPT OBJECTIVE: The student will multiply binomials.

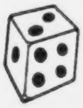
FOIL

PROCEDURE: Students can use the acronym, FOIL, to remember the process used to multiply binomials. The FOIL method consists of multiplying terms in the following order: First, Outer, Inner, and Last. A pictorial representation of the method will also show students the use of the distributive property.



POLYNOMIALS

CONCEPT OBJECTIVE: The student will practice multiplying binomials and evaluating expressions.



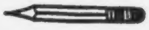
THE GREATEST VALUE

PROCEDURE: Each player needs an individual game sheet and pencil. Two or more players may participate. The leader needs a die. Before the game begins, the leader announces the value to be used to evaluate the polynomial. The leader rolls the die and calls out the numeral shown. Each individual player decides in which shape to place this numeral. Once the numeral is placed in a shape, the player may not change it.

The process continues three more times with the leader rolling the die and each player entering each numeral on his/her game sheet. After the four numerals have been determined, each player computes the product of the expression. The polynomial is then evaluated using the given value for the variable. The player having the greatest value, by using his/her polynomial, is the winner.

Reproducible game sheets may be found in the appendix.

CONCEPT OBJECTIVE: The student will practice factoring and multiplying polynomials.



FLASH CARDS

PROCEDURE: Flash cards may be used to practice factoring and multiplying polynomials. One side of each card would contain a polynomial while the other side of the same card would have the factored form of that polynomial. The flash cards may be used by an individual student or by a pair of students, checking each other's responses.

POLYNOMIALS

CONCEPT OBJECTIVE: The student will practice factoring and simplifying algebraic expressions.

SPELLING WITH ALGEBRA

PROCEDURE: Students can simplify the algebraic expressions to spell a word or word phrase. The letter, "O", is not used in any of the following to avoid confusion with zero.

1. $$\left[\frac{4S^2+4S+4}{x^2} - \frac{4(1+S^2)}{x^2}\right]\left[\frac{2MV}{2}\right]\left[\frac{I}{W} + \frac{4SMXY}{4YXMS} - 1\right]\left[\frac{X^2LEW}{4V}\right]$$

2. $$\left[\frac{(20AX)(S^3U^2R^2)(ISPE-1)}{X(A^3+2A^2B+AB^2)(X+Y)^2(10S^2U)}\right]\left[\frac{X^3Y+2X^2Y^2+XY^3}{(A-B)(A^2+AB+B^2)}\right]\left[\frac{(A^3-B^3)(A+B)^2}{2XY}\right] + SUR$$

3. $$RS(PING)^2\left\{\frac{\frac{A^2B^2+4(AB+1)}{GRNISP}\left[\frac{SIPNGR+2(SIPNGR)(E^2VRF)+RE^2FV}{(2+AB)^2}\right] - 2FE^2RV}{\frac{(PING-FR)(PING+FR)+(FR)^2}{PING}}\right\}$$

POLYNOMIALS

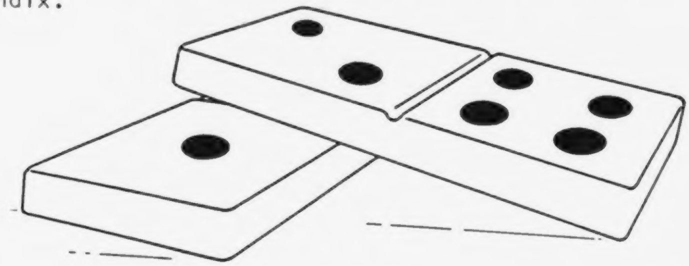
CONCEPT OBJECTIVE: The student will practice factoring and multiplying polynomials.

FACTORING DOMINOES

PROCEDURE: Each pair of students will need a set of dominoes. After all dominoes have been mixed face down, each player selects five. One domino is then selected from the pile and placed face-up in the center of the table to serve as the starter. Each player will be trying to match a polynomial with the factored form of that polynomial.

The first player tries to match one of his/her dominoes to the starter. If a match cannot be made, the player draws a domino from the pile and play passes to the opponent. For each match made, a player scores one point. When no more matches are possible for either player, the game ends. The player with the most points is the winner.

A sample list of factoring dominoes may be found in the appendix.



CONCEPT OBJECTIVE: The student will practice factoring and multiplying polynomials.

POLYNOMIAL RUMMY

PROCEDURE: *Algebra II, page 99.

GRAPHING

CONCEPT OBJECTIVE: The student will practice graphing linear equations.

ALPHABET

PROCEDURE: Each of the letters of the alphabet can be graphed in the coordinate plane. Directions may be in the form of linear equations with restrictions upon certain variables. Graphing the following will construct the letter, M.

$x = -8$ and $-9 \le y \le 9$

$x = 8$ and $-9 \le y \le 9$

$9x + 8y = 0$ and $-8 \le x \le 0$

$9x - 8y = 0$ and $0 \le x \le 8$

Equations with restrictions can be written to graph other letters of the alphabet.

CONCEPT OBJECTIVE: The student will determine if a relation is a function.

FUNCTIONS

PROCEDURE: Given a graphed relation, a yardstick can be used as a device to determine if the relation is a function. The yardstick should be placed on top of the graph, perpendicular to the x-axis. As it is moved, students should check to see that the yardstick does not intersect the graph in more than one point for any location along the graph. If the yardstick intersects the graph in two or more points at the same time, the relation is not a function.

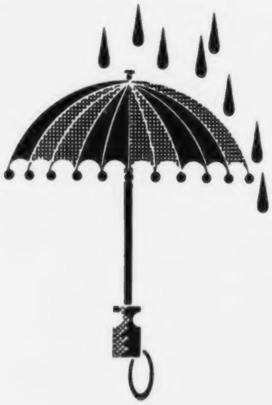
GRAPHING

CONCEPT OBJECTIVE: The student will practice graphing line segments given a point and the slope.

AN UMBRELLA GRAPH

PROCEDURE: Students can graph the following set of line segments by plotting the given points and sketching the line through that point with the given slope and applying the stated restrictions. Coordinate axes need to be constructed so that $-10 \leq x \leq 10$ and $-13 \leq y \leq 8$.

1.	(1,0)	$m = -1/2$	$-1 \leq x \leq 1$
2.	(-3,0)	$m = 1/2$	$-5 \leq x \leq -1$
3.	(-7,1)	$m = -1$	$-1 \leq y \leq 1$
4.	(-10,0)	$m = 1/3$	$-10 \leq x \leq -7$
5.	(-8,3)	$m = 3/2$	$0 \leq y \leq 6$
6.	(0,8)	$m = 1/3$	$6 \leq y \leq 8$
7.	(7,6)	$m = -2/7$	$6 \leq y \leq 8$
8.	(9,2)	$m = -2$	$7 \leq x \leq 10$
9.	(6,1)	$m = -1/4$	$6 \leq x \leq 10$
10.	(4,0)	$m = 1/2$	$-1 \leq y \leq 1$
11.	(1,0)	$m = -1$	$-1 \leq y \leq 0$
12.	(2,-12)	$m = -12$	$1 \leq x \leq 2$
13.	(1,-13)	$m = 1$	$-13 \leq y \leq -12$
14.	(2,-13)	$m = 0$	$0 \leq x \leq 1$
15.	(-1,-12)	$m = -1$	$-1 \leq x \leq 0$



SYSTEMS OF EQUATIONS

CONCEPT OBJECTIVE: The student will practice solving systems of linear equations.

STAR SYSTEMS

PROCEDURE: Students can solve the following systems of linear equations. Solutions can be written as coordinates in the form (x,y) and plotted on a set of corrdinate axes. Connecting the solution sets in order, as they are plotted, will produce a recognizable figure.

1. $x = y - 4$
   $3x + y = 4$

2. $x = -5$
   $y - x = 12$

3. $\frac{1}{2}(x+y) = -1$
   $-x + y = 4$

4. $-2x = -3y + 7$
   $-2y - x = 14$

5. $x - y = 1$
   $2x + 4y = -13$

6. $\frac{x}{2} + \frac{y}{4} = \frac{-3}{2}$
   $\frac{x}{4} - \frac{y}{2} = \frac{17}{4}$

7. $2x - y = 7$
   $y = 4x - 12$

8. $x + 3y = 3$
   $x - 2y = 13$

9. $2x + y = 10$
   $3x - 2y = 8$

10. $3x - y = 10$
    $2x + y = 20$

CONCEPT OBJECTIVE: The student will practice solving a system of equations.

BASKETBALL COURT

PROCEDURE: *Algebra II, page 96.

SYSTEMS OF EQUATIONS

CONCEPT OBJECTIVE: The student will practice solving a system of equations.

SYSTEMS RELAY

PROCEDURE: Each team of four students is seated in a column of 4 desks, one behind the other. As the relay begins, each student in the team receives a system of two equations. The students in the second, third, and fourth positions receive systems of two equations in three variables and are dependent upon their teammates for information before they can solve their problems.

The first student receives a system of 2 equations in two unknowns and as quickly as possible, solves for the two variables and hands the solution to the player seated behind in the column. The second student uses the information received to solve his/her system of equations. The new solutions are added to the information sheet from the first player and sent to the third player. The process continues until the fourth player has solved the last pair of equations. As play continues, each player should check his/her work. If an error is found, a new solution should be sent back to the team. The winner of the relay race is the first team to get a complete correct solution for their problems.

An example of relay problems is given below.

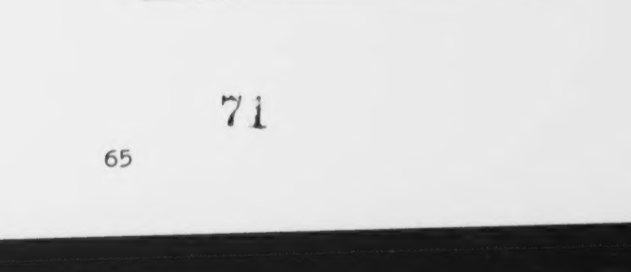
I. $a + b = 3$ $3a - 5b = 17$

II. $2a + 3x - y = 11$ $x - 5b + 3y = 16$

III. $2r + 5s - 4y = -1$ $3r + 3x - 2s = -6$

IV. $5r + 3c + 4d = -35$ $-3d + 2c - 6s = -12$

GEOMETRY
PROVING THEOREMS, WORKING PROBLEMS
HELPING US TO THINK.



GEOMETRY

The study of geometry uses a set of definitions and postulates to develop a deductive system. Applications of the properties of triangles, circles, and other geometric figures provide the student with an awareness of the usefulness of geometric concepts. Trigonometric ratios and geometric solids may be introduced. The structure and solution of proofs is a major emphasis, giving the student experience in logical reasoning.

TOPIC	PAGE
PROOFS	69
SIMILAR POLYGONS	72
RIGHT TRIANGLES	73
CIRCLES	76
SOLIDS	79
CONSTRUCTIONS	85

The contents of this chapter are organized by four strands coded with symbols as follows:

✏ - Activity
🌎 - Application
🎲 - Game
🔍 - Problem-solving Experience

Many activities are coded under two or more strands.

PROOFS

CONCEPT OBJECTIVE: The student will practice constructing two-column proofs.

100 PROOF GIN

PROCEDURE: Two teams, each composed of two students, will compete against each other. For each game, players will need three kinds of cards. One set of cards contains STATEMENTS, another set contains REASONS, and the third set will be PROOFS, statements to prove. All 3 decks of cards are shuffled and placed face down.

To begin, each team draws one card from the stack of PROOFS. This card will have the proof they will be attempting to construct. Students may want to use some means to keep their opponents from seeing their cards. One team begins play by drawing one card from the STATEMENTS pile and one card from the REASONS pile. They may choose to keep one or both of the cards or discard one or both of them. Discards are placed face up next to the appropriate stack. The goal for each team is to construct a logical correct 2-column proof using STATEMENT and REASON cards.

In a turn, each team may select one STATEMENT card from the stack or the top card from the discard pile and one REASON card from the stack or the top card from the discard pile.

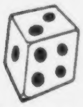
If all cards in a stack have been used before one team has solved their proof, the cards in the discard pile should be reshuffled and placed face down to form a new stack. The first team to correctly construct their proof wins the game. If a team has done their proof incorrectly, their opponents win the game. More than 2 teams or individuals can play. A solitaire version can also be played.

A sample listing of PROOF, STATEMENT, and REASON cards may be found in the appendix.

74

PROOFS

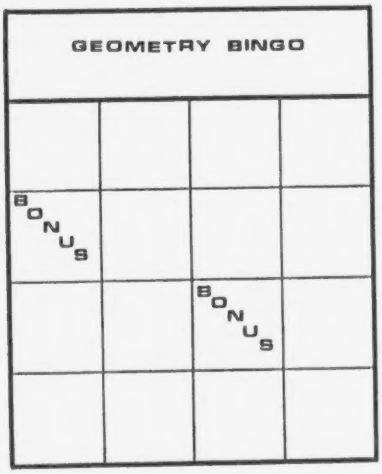
CONCEPT OBJECTIVE: The student will review basic geometric terms and properties used in proofs.



GEOMETRY BINGO

PROCEDURE: Each student needs a game board and markers. The leader needs a set of question cards related to the game boards. In each square of the game board an illustration, definition, theorem, or postulate relating to various geometric concepts will have been written. The teacher begins the game by selecting a card and reading the question. Each student may cover the appropriate square on his/her game board. The first student to cover all of the squares in a row, column, or diagonal is the winner. If it is determined that the winner has covered his/her board incorrectly, the player must forfeit the game and play continues until a new winner is determined.

Game boards may consist of any number of squares. In creating the question cards, it will be helpful to have more cards than game board squares. Different game boards can be created by changing the arrangement of the squares. A sample 4x4 game board and question cards may be found in the appendix.



PROOFS

CONCEPT OBJECTIVE: The student will practice constructing valid proofs.

5 = 0

PROCEDURE: Students can determine the error made in the following proof.

Given: $x = 5$

Prove: $5 = 0$

STATEMENTS	REASONS
1. $x = 5$	Given
2. $x^2 = 5x$	Multiplication property of equality
3. $x^2 - 5x = 0$	Subtraction property of equality
4. $x(x - 5) = 0$	Distributive property
5. $x = 0$	Division property of equality
6. $5 = 0$	Substitution

SIMILAR POLYGONS

CONCEPT OBJECTIVE: The student will use similar triangles to solve problems.

ME AND MY SHADOW

PROCEDURE: When the sun is shining outside, students may enjoy using the ancient Greek method for measuring heights using shadow length. A student can hold a stick of a known length perpendicular to the ground while another measures the length of the shadow it casts. The student can then select another object and measure the length of its shadow. A proportion can be used to determine the unknown height. Both shadow measurements need to be taken within a few minutes of each other.

The students' heights can also be found using this method and then compared with their actual heights.

CONCEPT OBJECTIVE: The student will use the concepts of similarity to solve problems.

PHOTO REDUCTION

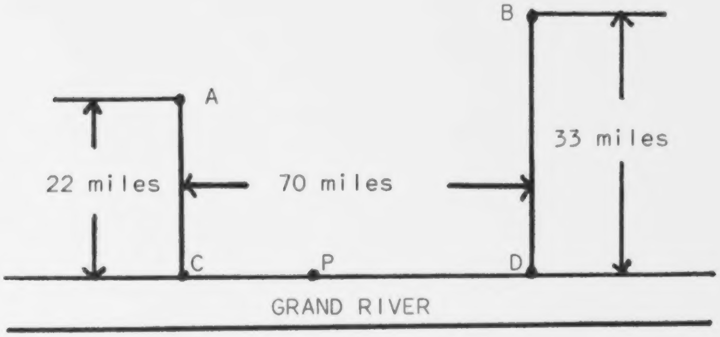
PROCEDURE: A printer is preparing a negative of a photo for a print plate. When an order calls for a reduction, the camera must be placed closer than the original distance. At the camera's original distance of 5 feet, the length of the photo is 12 inches. Determine the length of the photo if the camera is placed at a distance of 6.25 feet.

SIMILAR POLYGONS

CONCEPT OBJECTIVE: The student will use similar triangles to solve problems.

WATER PIPELINE

PROCEDURE: The Grand River Project has water storage towers at two locations, A and B. A water treatment plant is to be built on the banks of the Grand River. Pipelines must be installed from the storage towers to the treatment plant. Decide where to locate the treatment plant, P, so that the total pipeline length from A to the plant and from B to the plant will be at a minimum.



RIGHT TRIANGLES

CONCEPT OBJECTIVE: The student will prove the Pythagorean Theorem.

PYTHAGOREAN THEOREM

PROCEDURE: There are numerous proofs of the Pythagorean Theorem. Students can be encouraged to construct their own. Concrete models and area formulas for various geometric figures can be useful.

RIGHT TRIANGLES

CONCEPT OBJECTIVE: The student will be guided to derive the distance formula.

DISTANCE FORMULA

PROCEDURE: Students can review that the distance between two points on a horizontal or vertical number line can be found by counting the units between the points. Finding the length of a segment which is neither horizontal or vertical will involve an application of the Pythagorean Theorem.

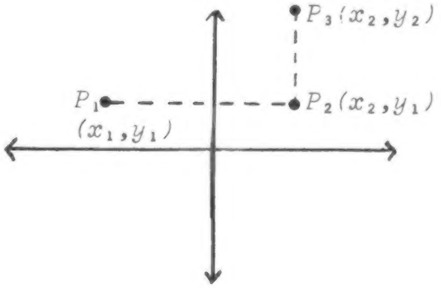
Three points can be located on a set of coordinate axes similar to the following diagram and the distances between all three points can be found. The distance between P_1 and P_2 is $|x_1 - x_2|$ and the distance between P_2 and P_3 is $|y_1 - y_2|$. Since the three points form a right triangle, the distance between P_1 and P_3 can be found using the Pythagorean Theorem.

$$(P_1P_3)^2 = (P_1P_2)^2 + (P_2P_3)^2$$

$$= |x_1 - x_2|^2 + |y_1 - y_2|^2$$

$$= (x_1 - x_2)^2 + (y_1 - y_2)^2$$

$$P_1P_3 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$



RIGHT TRIANGLES

CONCEPT OBJECTIVE: The student will practice using the distance formula.

THE GREATEST DISTANCE

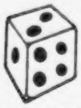
PROCEDURE: A group of 2-6 players will need a set of 30-40 cards on which ordered pairs have been written. Ordered pairs may be randomly selected such that $-10 \le x \le 10, -10 \le y \le 10$, and $x, y \in \{\text{integers}\}$. After shuffling, the dealer deals each player 2 cards. The remaining cards are placed face down in a stack by the dealer. Players look at their cards individually and compute the distance between the points represented on their cards. Play begins with the person on the left of the dealer. In turn, a player may choose to trade one card for the top one of the stack or choose to keep the original two cards. After each player has had a chance to trade, all players reveal the value of the distance between their two points. The player with the greatest distance value wins the round and receives one point.

In case of a tie, those players each receive a point. If a player computes the distance value incorrectly, the player with the next highest value wins the round. At the end of a round all cards are shuffled. The deal passes to the next person in a clockwise direction. At the end of the time period, the player with the most points wins the game.

THE GREATEST DISTANCE

CIRCLES

CONCEPT OBJECTIVE: The student will practice identifying equations, graphs, and characteristics of circles.

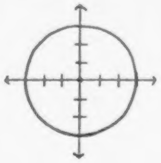
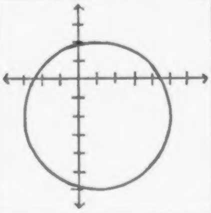


CIRCLE THREE

PROCEDURE: Each group of 2-4 players needs a deck of Circle Three cards. After shuffling, the dealer deals 9 cards to each player. The rest of the deck is placed face down to form a stack. The top card is turned over and placed next to the stack to start a discard pile. Players look at their hands individually for any sets of three matching cards. A matching set consists of an equation card, description card, and graph card describing the same circle.

After matching sets, if any, have been removed from each player's hand and placed on the table, play begins with the person on the left of the dealer. In turn, players may select the top card of the stack or the top card of the discard pile. Players must discard at the end of their turns. The first of the players to match all of their cards is the winner. If all of the stack has been used before a winner is determined, the top card can be used to start a new discard pile, and the rest of the cards shuffled to form a new stack.

A deck of Circle Three cards may consist of 15-20 sets of three matching cards. Each set of three cards must have equivalent equation, description, and graph cards. Two examples of three matching cards are given below.

$x^2 + y^2 = 9$	CENTER (0,0) RADIUS = 3	
$(x-1)^2+(y+2)^2 = 16$	CENTER (1,-2) RADIUS = 4	

CIRCLES

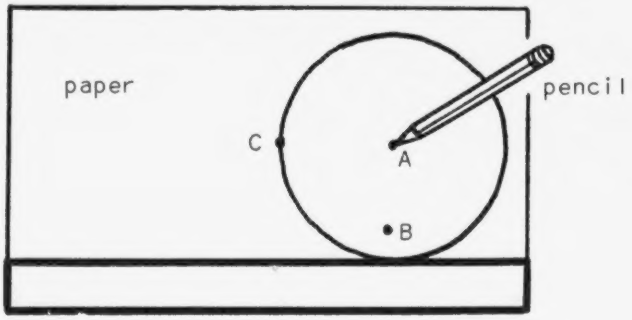
CONCEPT OBJECTIVE: The student will investigate the path of various points on and inside a circle.

POINTS ON A WHEEL

PROCEDURE: There are an infinite number of points on a bicycle wheel. To determine if any point actually makes a circle as it goes around, the following experiment can be made. A model of a wheel can be made by carefully cutting a circle out of poster board or cardboard.

Three kinds of points, the center, points on the circle, and points inside the circle, can be identified. A hole can be made in the center of the circle. Placing a sharp pencil point in the hole, the circle can be carefully rolled along a straight edge. The path of the center point will be traced by the pencil.

Next, a point other than the center can be selected inside the circle and a hole made for that point. Again, the pencil point can be placed in the hole and the circle rolled along the straight edge.



Lastly, a point on the rim of the circle model can be selected and the experiment repeated. The student may make some conclusions about the paths traced from the various points.

CIRCLES

CONCEPT OBJECTIVE: The student will be guided through the development of the formula for the area of a circle.

AREA OF A CIRCLE

PROCEDURE: *General Math, page 30.

CONCEPT OBJECTIVE: The student will inscribe a pentagon.

INSCRIBING A PENTAGON

PROCEDURE: Students can follow these directions to inscribe a pentagon.

1. Draw two circles with the same radius.
2. In the first circle, draw diameter $\overline{AB}$.
3. Construct $\overline{CD}$ as a perpendicular bisector of $\overline{AB}$, where C is a point on the circle and D is the center of the circle.
4. Bisect $\overline{DB}$. Label its midpoint M.
5. Using M as the center and $\overline{CM}$ as a radius, draw an arc intersecting $\overline{AD}$ at E.
6. CE is the required length of one side of the inscribed pentagon.
7. On the second circle, mark a beginning point. With a ruler, use CE as the length of one side and inscribe a pentagon in the second circle.

SOLIDS

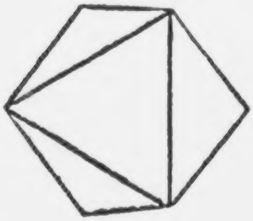
CONCEPT OBJECTIVE: The student will use paper models to construct Platonic solids.

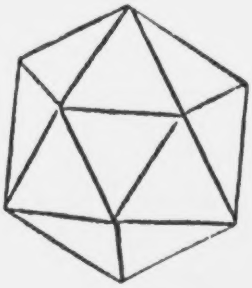
PLATONIC SOLIDS

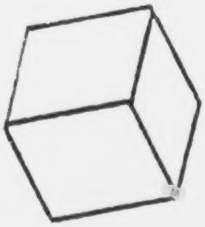
PROCEDURE: There are five regular polyhedra characterized with (1) all edges having the same length; (2) all faces having the same size and shape; and (3) the same number of edges meeting at each corner. They are sometimes referred to as Platonic solids, named after Plato, who gave instructions for making models of them.

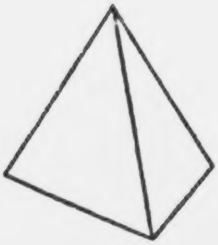
A pattern for each polyhedron may be found on the following pages. The student should cut on the solid lines and fold inward on the dashed lines.

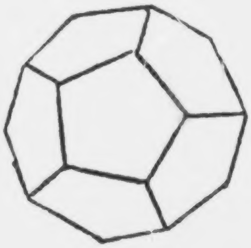
There are 13 Archimedean solids, various prisms, and other regular and irregular polyhedra. Students can be encouraged to construct a model of an existing polyhedra or create their own.



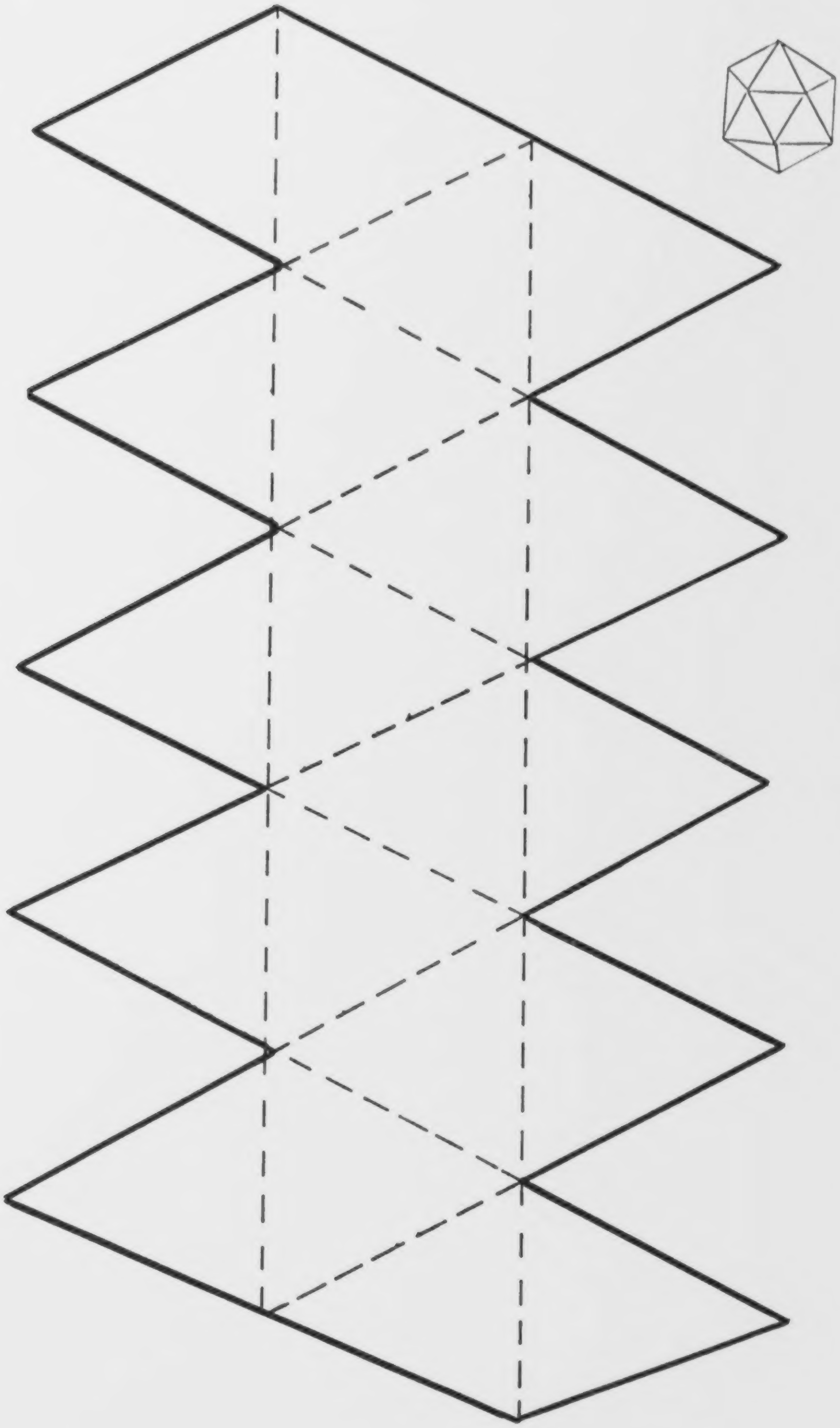




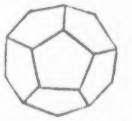


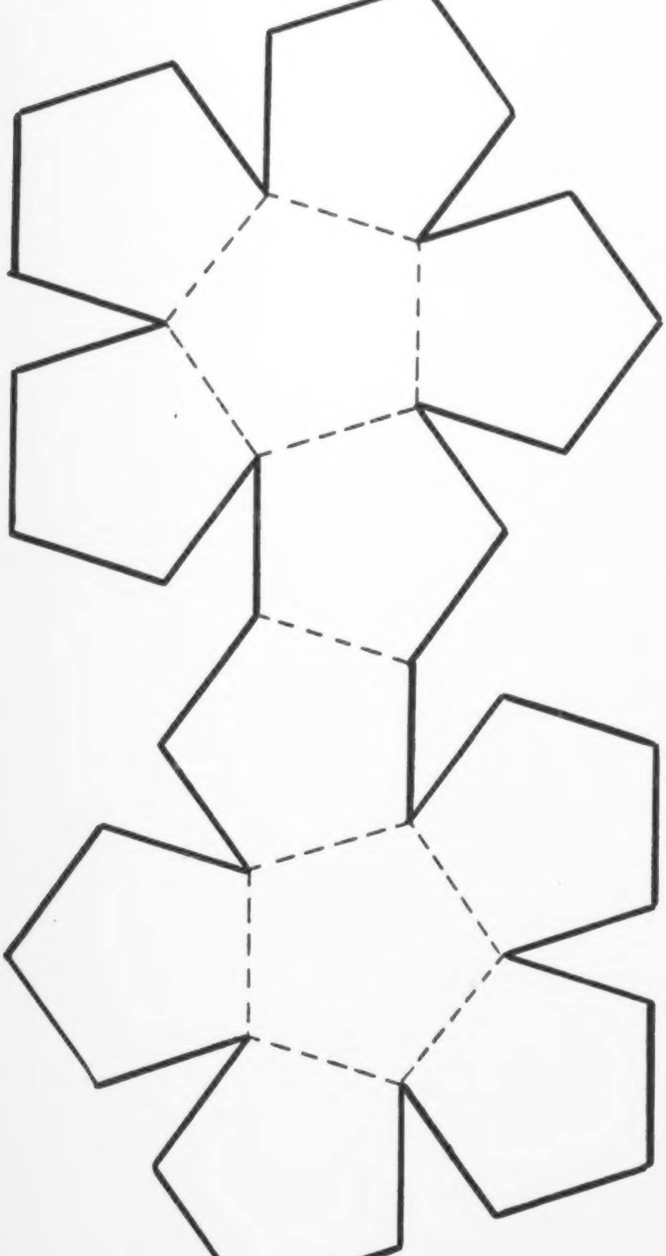


ICOSAHEDRON

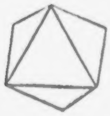


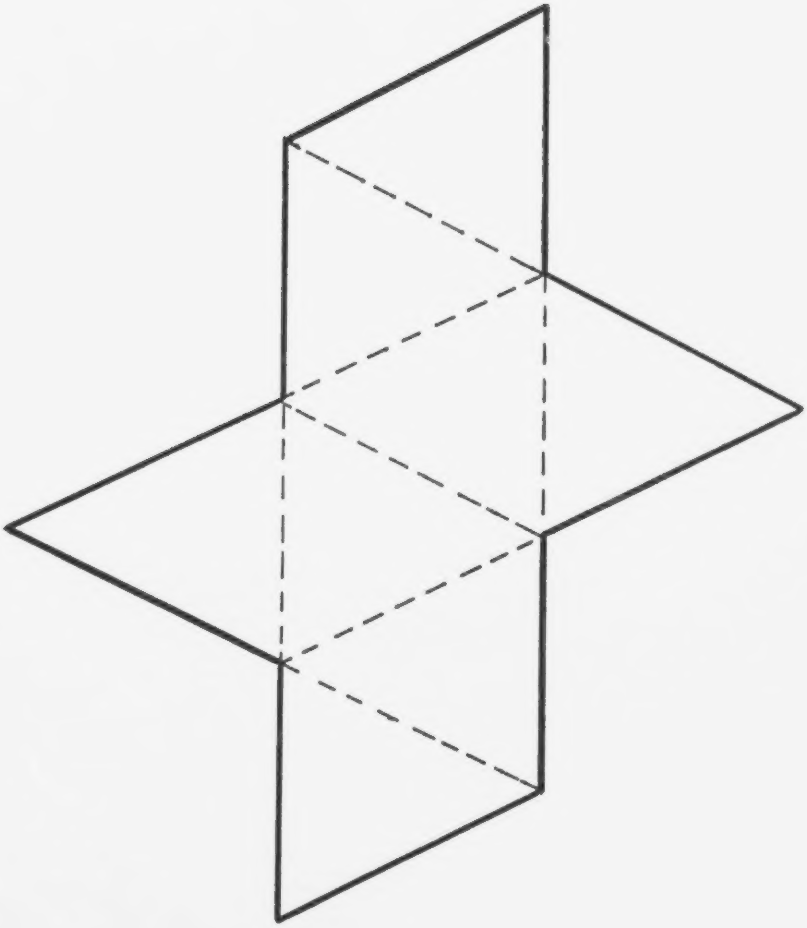
DODECAHEDRON



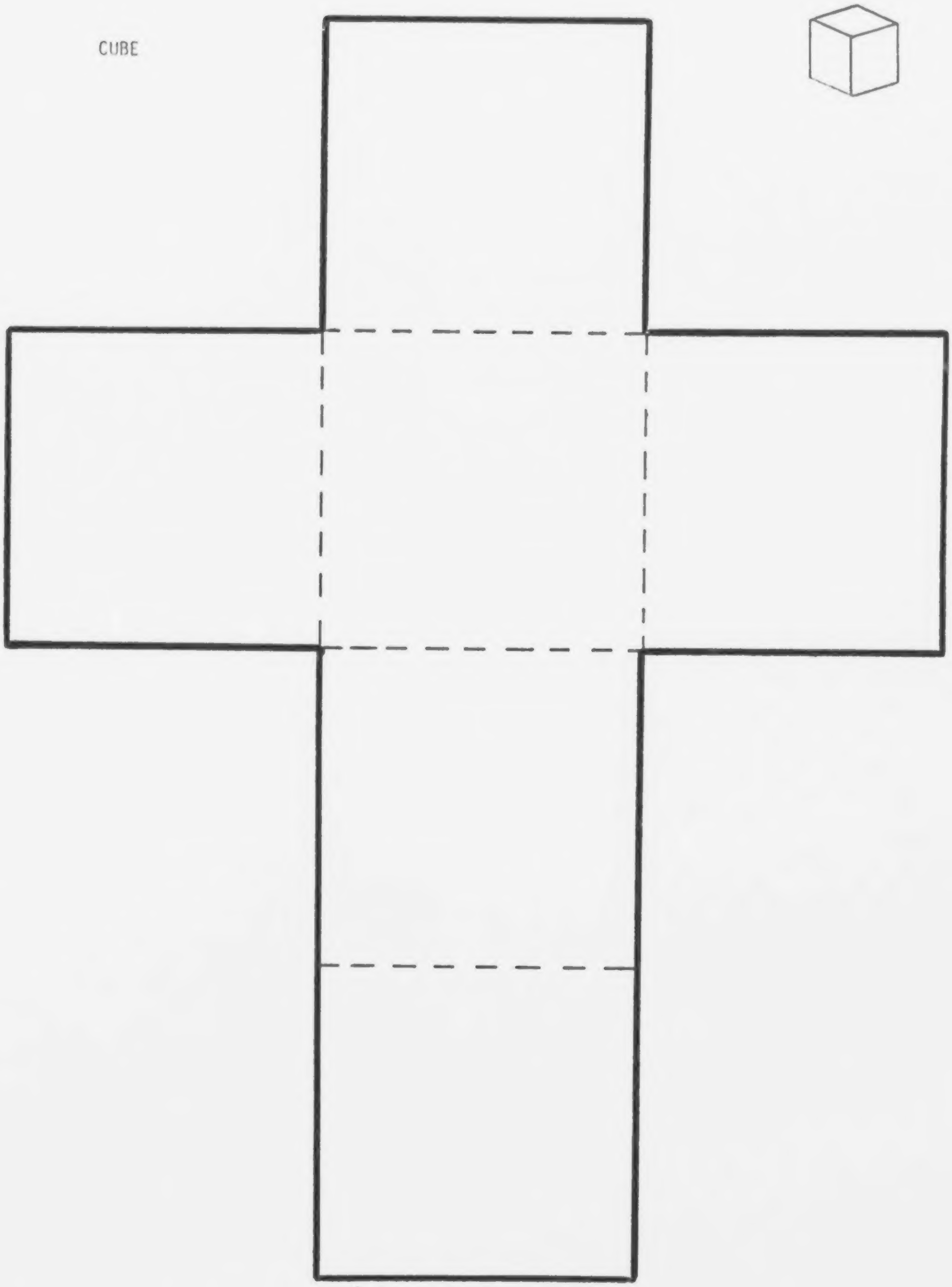


OCTAHEDRON

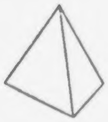


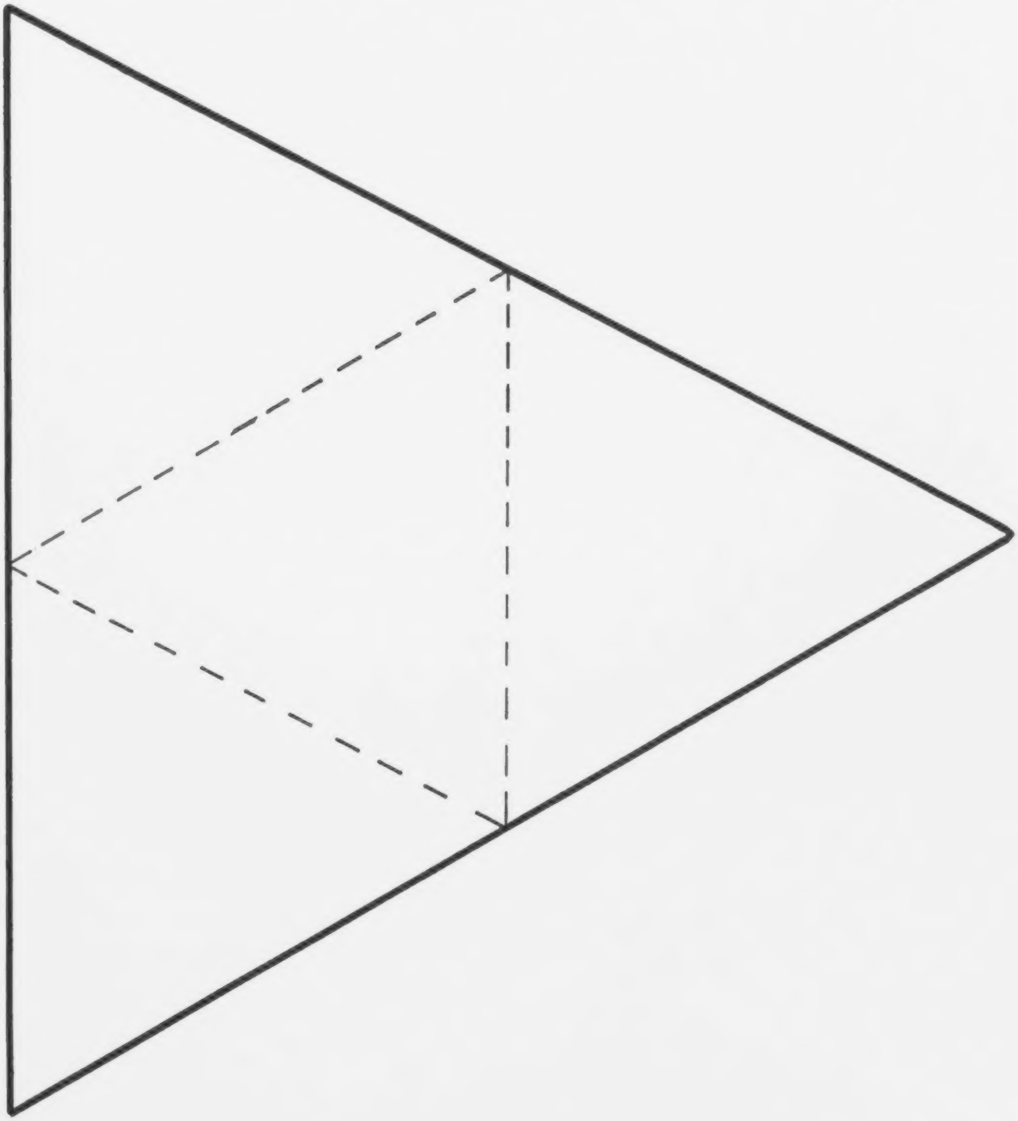


CUBE



TETRAHEDRON





SOLIDS

CONCEPT OBJECTIVE: The student will investigate some of the properties of regular polyhedra.

PAINTING POLYHEDRA

PROCEDURE: Using models of the five regular polyhedra the student can determine the number of faces, vertices and edges for each polyhedron. Suppose the faces of each polyhedron are to be painted so that each face will be painted completely with a single color and no two adjacent faces will have the same color. Students can determine the least number of colors needed to paint each polyhedron.

Similar investigations can be made if the vertices of each solid are painted so that no two vertices connected by an edge have the same color or if the edges are painted so that no two edges meeting at the same vertex have the same color.

CONSTRUCTIONS

CONCEPT OBJECTIVE: The student will practice basic constructions.

TANGRAM PUZZLE

PROCEDURE: Students can use the following directions to construct the seven piece tangram puzzle in the form of a large square.

1. Draw a 10 cm square. Label it ABCD.
2. Bisect $\overline{AB}$ and label the midpoint E.
3. Bisect $\overline{AD}$ and label the midpoint F.
4. Draw $\overline{FE}$.
5. Draw $\overline{DB}$.
6. Bisect $\overline{EF}$ and label the midpoint G.
7. Draw $\overline{CG}$.
8. Label the intersection point of $\overline{GC}$ and $\overline{BD}$ as H.
9. Construct $\overline{EJ} \perp \overline{BH}$ so that $\overline{HJ} = \overline{JB}$.
10. Bisect $\overline{DH}$ and label the midpoint K.
11. Draw $\overline{KG}$.

CONSTRUCTIONS

CONCEPT OBJECTIVE: The student will investigate some properties of angle and segment bisectors.

PAPER FOLDING

PROCEDURE: Each student will need a plain sheet of paper. In the following directions the student should draw large figures so that the concepts can be clearly observed.

1. On the paper, draw an angle. Then fold the paper, one side of the angle onto the other, and crease it. The fold represents an angle bisector.
2. Draw a triangle and fold the paper to create the three angle bisectors. The student can record observations about the three segments.
3. On the paper, draw a line segment. Then fold the paper, one endpoint onto the other, and crease it. The fold represents the perpendicular bisector of the segment.
4. Draw a triangle and fold the paper to create the three perpendicular bisectors of the sides. The student can record observations about the three segments.

CONSTRUCTIONS

CONCEPT OBJECTIVE: The student will use wax paper to create geometric figures.

WAX PAPER CONSTRUCTION

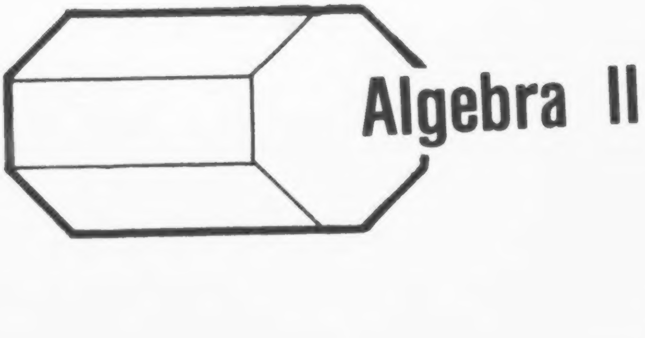
PROCEDURE: Each student will need a compass and a piece of wax paper in the shape of a square or rectangle. Place a dot in the center of the paper and draw a circle using the dot as the center. Be careful not to pierce the wax paper.

Fold the wax paper so that the curve of the circle is on top of the dot and crease the paper. Move or rotate the curve to the left about ¼" keeping the curve on top of the dot and crease the paper again. Keep folding the paper until all of the circle has passed through the dot. The result should be another circle inside and concentric with the original circle. Paper folding exercises may be used to create other geometric figures such as parabolas and hyperbolas.

ALGEBRA TWO

SYSTEMS, GRAPHS, WORD PROBLEMS

PREPARING FOR THE FUTURE.



ALGEBRA II

Algebra II extends the student's experience with functions and relations as the properties of linear, quadratic, exponential, and logarithmic functions are explored. The study of the real number system incorporates irrational numbers, and the concept of complex numbers is introduced. Applications of polynomials and systems of equations and inequalities help familiarize the student with the usefulness of algebra in real-life situations. Word problems which require algebraic solutions help students develop problem-solving skills.

TOPIC	PAGE
EQUATIONS	93
SYSTEMS OF OPEN SENTENCES	96
POLYNOMIALS	99
FUNCTIONS AND RELATIONS	104
GRAPHING	111
SERIES	112

The contents of this chapter are organized by four strands coded with symbols as follows:

✏ - Activity
🎲 - Game
🌎 - Application
🔍 - Problem-solving Experience

Many activities are coded under two or more strands.

EQUATIONS

CONCEPT OBJECTIVE: The student will solve fractional equations.

BANANA BREAD

PROCEDURE: Students can determine the following recipe by solving each equation. Some may want to check their work by baking and tasting.

1. Cream y cups of butter or margarine and x cups of sugar in one bowl, where y and x are respective solutions for the equations

   $$\frac{1}{5} + \frac{1}{y} = \frac{6}{5} \qquad \frac{1 + x}{x} = \frac{3}{x}$$

2. Beat w eggs until light, where w is the solution for the equation

   $$\frac{5}{w} - \frac{2}{3} = \frac{7}{12}$$

   Add eggs to the creamed mixture.

3. Mash n bananas thoroughly with a fork and blend with the mixture, where n is the opposite of the solution for the equation

   $$\frac{n}{2n + 4} - \frac{1}{n + 2} = 1$$

4. Sift d cups of flour, k teaspoons of soda, and m teaspoons of salt together and mix quickly with the other ingredients. The value of d is the square of the solution to the equation

   $$\frac{d + 3}{d - 1} + \frac{d + 1}{d - 3} = 2$$

   The value of k is the integral solution to the equation

   $$\frac{3}{k - 1} + \frac{2}{k} = 4$$

   The value of m is the opposite of the multiplicative inverse of the solution for the equation

   $$\frac{4}{m^2 - 4m} - \frac{m - 3}{m - 4} = \frac{1}{m}$$

5. Separate mixture into z equal quantities and place in loaf pans, where z is the solution for the equation

   $$\frac{z - 1}{z} - \frac{z - 3}{z - 6} = \frac{1}{z}$$

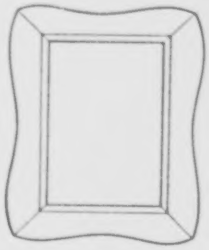
6. Bake for e minutes at r degrees where e is twice the product of the first three prime numbers and r is one more than the square of 18.

EQUATIONS

CONCEPT OBJECTIVE: The student will practice solving problems using a linear equation in two variables.

PICTURE FRAMES

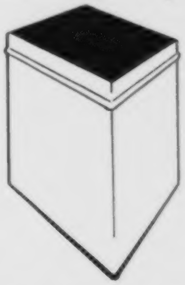
PROCEDURE: A manufacturer of picture frames ships cartons which contain either eight 5"x7" frames or four 8"x10" frames. He will ship no partial cartons. The wholesale price of a 5"x7" frame is $4.50 and the price of an 8"x10" frame is $6.00. A retail outlet has a $600 contract for the two items of merchandise. What choices of quantities does the retailer have to exactly fill the contract?



CONCEPT OBJECTIVE: The student will practice solving quadratic equations.

A TIN BOX

PROCEDURE: A square piece of tin is to be used to construct an open box having a volume of 162 cubic inches. The box is to be constructed by cutting 2-inch squares from each corner and turning up the sides. Determine the dimensions required for the piece of tin.



CONCEPT OBJECTIVE: The student will practice solving problems using equations.

OLD TEX'S GOLD

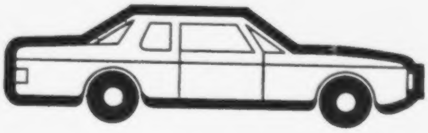
PROCEDURE: *Algebra I, page 51.

EQUATIONS

CONCEPT OBJECTIVE: The student will demonstrate the concepts present in rate-time-distance problems.

TRAVELING

PROCEDURE: *Algebra I, page 53.



CONCEPT OBJECTIVE: The student will practice logical thinking.

3 = -3

PROCEDURE: The following proof concludes that 3 is equal to -3. Students can be encouraged to determine what is wrong with the reasoning.

1. Let $x = 3$
2. Then $x^2 = 9$
3. $x^2 - 9 = 9 - 9$
4. $x^2 - 9 = 0$
5. $(x + 3)(x - 3) = 0$
6. $\frac{(x + 3)(x - 3)}{(x - 3)} = \frac{0}{(x - 3)}$
7. $(x + 3) = 0$
8. $x = -3$
9. Therefore, $3 = -3$

SYSTEMS OF OPEN SENTENCES

CONCEPT OBJECTIVE: The student will practice solving problems using a system of equations.

BASKETBALL COURT

PROCEDURE: Students can measure the school basketball court to determine the length and width. The National Federation Basketball Rule Book cites the dimensions of an ideal court for high school play. Given that the perimeter and area of the court are 268 feet and 4200 square feet, respectively, determine the dimensions. Students can compare their school court dimensions, perimeter, and area with those of the ideal court.

CONCEPT OBJECTIVE: The student will practice solving systems of linear equations.

STAR SYSTEMS

PROCEDURE: *Algebra I, page 62.

CONCEPT OBJECTIVE: The student will practice solving a system of equations.

SYSTEMS RELAY

PROCEDURE: *Algebra I, page 63.

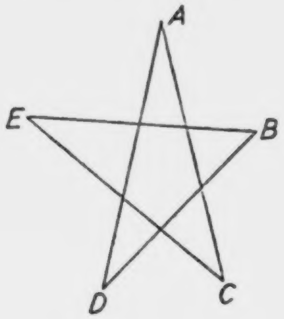
SYSTEMS OF OPEN SENTENCES

CONCEPT OBJECTIVE: The student will practice solving a system of linear equations.

STAR SUM

PROCEDURE: In the given figure, find the sum:

$m\measuredangle A + m\measuredangle B + m\measuredangle C + m\measuredangle D + m\measuredangle E$



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SYSTEMS OF OPEN SENTENCES

CONCEPT OBJECTIVE: The student will solve a system of inequalities.

COURT CIRCLES

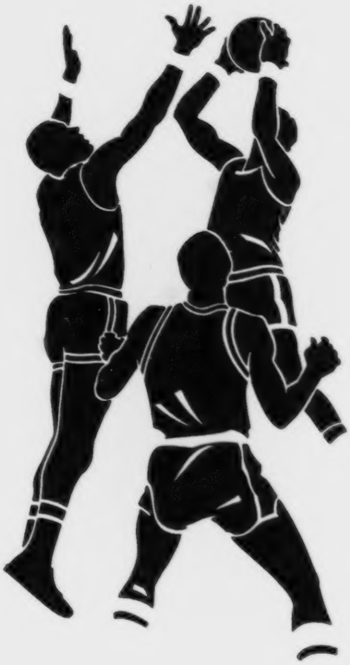
PROCEDURE: Students can measure the radius of each circle at the center of the school basketball court. An equation can be written for each circle using the center of the court as the origin, (0,0). The area between the two circles can be displayed graphically using two inequalities.

According to the National Federation Basketball Rule Book, the radii of the center court circles of an ideal high school court are 2 and 6 feet, respectively. The area between the circles can be represented graphically using the following inequalities.

$$x^2 + y^2 > 4$$

$$x^2 + y^2 < 36$$

Comparisons can be made between school court dimensions and those of an ideal court.



SYSTEMS OF OPEN SENTENCES

CONCEPT OBJECTIVE: The student will practice solving a system of inequalities.

BOXES OF CANDY

PROCEDURE: A candy manufacturer has 500 kg of chocolate, 100 kg of nuts, and 85 kg of fruit in her inventory. She produces two types of candy. A box of type A contains 3 kg of chocolate, 1 kg of nuts, and 1 kg of fruit and sells for $10. A box of type B, containing 5 kg of chocolate and 0.5 kg of nuts, sells for $6. Assuming that she can sell as much of each type as she produces, how many boxes of each kind of candy should she make to maximize her revenue?

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POLYNOMIALS

CONCEPT OBJECTIVE: The student will practice factoring and multiplying polynomials.

POLYNOMIAL RUMMY

PROCEDURE: Each group of 2-4 students will need a deck of Polynomial Rummy cards. After the cards are shuffled, each player is dealt 7 cards. The remaining cards are placed face down to form a stack and the top card of the stack is turned up to start a discard pile. The player to the left of the dealer begins play. A player may draw a card from the stack or take the top card from the discard pile. During a turn, a player may lay down 3 or more equivalent cards or 1 or more cards that match with any cards already laid down by the player or an opponent. Each player must discard at the end of a turn.

The round is over when one player wins by laying down all of the cards from his/her hand. If the stack is depleted before any player wins the round, the discard pile can be shuffled and used as a new stack. When the round is over, each player gets 1 point for each card laid down, and the winner of the round gets 3 extra points. One point is subtracted from a player's score for every card left in his/her hand. The first player to get 30 points is the winner of the game.

A sample list of Polynomial Rummy cards may be found in the appendix.

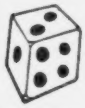
POLYNOMIALS

CONCEPT OBJECTIVE: The student will practice operations with polynomials.

TRY A DIGIT

PROCEDURE: *Algebra I, page 56.

CONCEPT OBJECTIVE: The student will practice multiplying binomials and evaluating expressions.



THE GREATEST VALUE

PROCEDURE: *Algebra I, page 57.

CONCEPT OBJECTIVE: The student will practice factoring and multiplying polynomials.

FLASH CARDS

PROCEDURE: *Algebra I, page 57.

POLYNOMIALS

CONCEPT OBJECTIVE: The student will practice factoring and simplifying algebraic expressions.

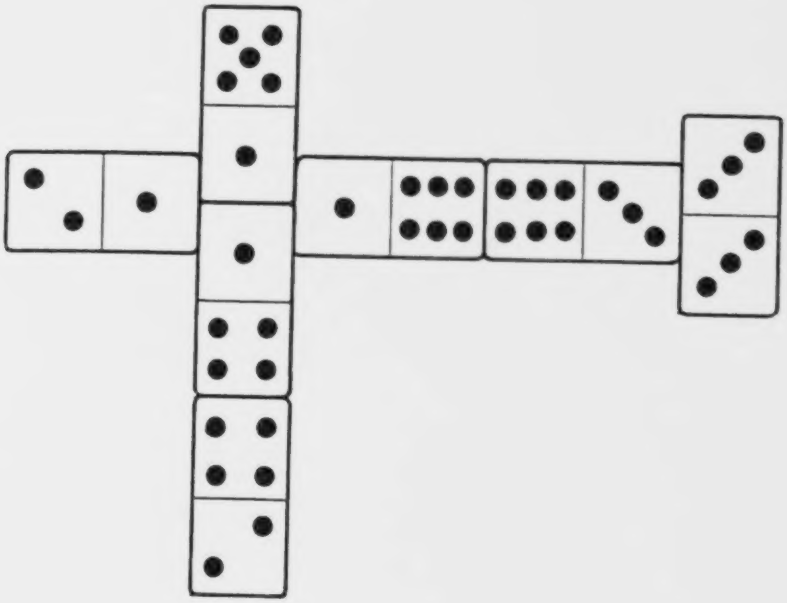
SPELLING WITH ALGEBRA

PROCEDURE: *Algebra I, page 58.

CONCEPT OBJECTIVE: The student will practice factoring and multiplying polynomials.

FACTORING DOMINOES

PROCEDURE: *Algebra I, page 59.



POLYNOMIALS

CONCEPT OBJECTIVE: The student will use factoring to solve inequalities.

SOLVING INEQUALITIES

PROCEDURE: Polynomial factoring and the use of a number line may help students solve some inequalities. All terms should be grouped on one side of the inequality and after factoring, a number line is constructed for each factor. On each number line, label only the zeroes of the factors of the polynomial. The locations on the number line where the factor will be positive are marked with plus signs and the places where the factor will be negative are marked with minus signs.

An additional number line is drawn to represent the product of all the factors. If the inequality contains the greater than symbol, then an analysis is made of the other number lines to determine the intervals where the product will be positive. The product will be positive whenever an even number of factors are negative or no factors are negative. The intervals for which the product is positive can be graphed as the solution set.

A similar analysis would be made if the original inequality contains the less than symbol. The product will be negative when an odd number of factors are negative. An example solution is shown on page 103.

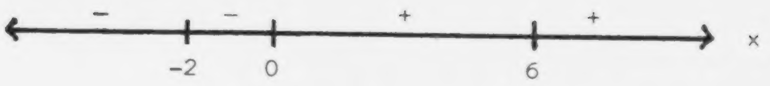
PROBLEM: Graph the solution set of $x^3 \leq 4x^2 + 12x$ over $\mathcal{R}$

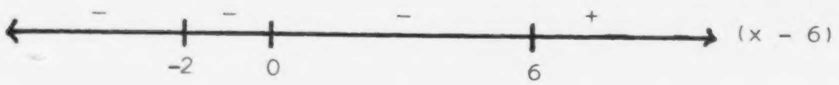
$$x^3 \leq 4x^2 + 12x$$

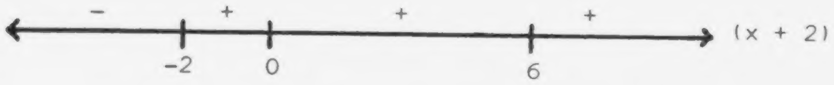
$$x^3 - 4x^2 - 12x \leq 0$$

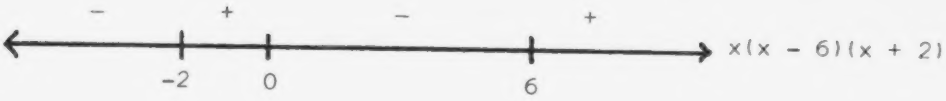
$$x(x^2 - 4x - 12) \leq 0$$

$$x(x - 6)(x + 2) \leq 0$$

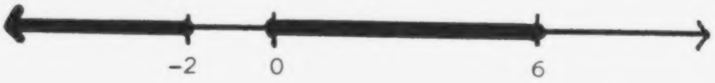








$\left\{x:\ x \leq -2\right\} \cup \left\{x:\ 0 \leq x \leq 6\right\}$



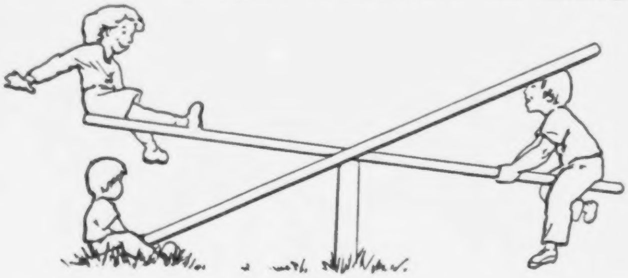
FUNCTIONS AND RELATIONS

CONCEPT OBJECTIVE: The student will experiment with an inverse variation.

SEESAW

PROCEDURE: A concrete example of inverse variation can be provided using the seesaw on a playground. Two students of different weights can be seated at opposite ends of the seesaw. The larger student should then move toward the center to achieve balance. Other students can measure the distance of each student from the center of the seesaw while also noting the weight of each participant. The experiment can be repeated several times using different students.

After collecting the data, students can be encouraged to find the relationship between the weights and distances which balanced the seesaw. The conclusion that $w_1d_1 = w_2d_2$ may be reached with students observing that as one value increases the other decreases to maintain the constant product.



CONCEPT OBJECTIVE: The student will use inverse variation to solve problems.

BOYLE'S LAW

PROCEDURE: Boyle's Law from physics offers an example of inverse variation. The formula states that when the temperature of a gas is constant, $P = \frac{k}{v}$ where P, the pressure, varies inversely as the volume occupied by the gas. A specific application has some oxygen occupying 450in^3 at a pressure of 30.0 lb/in^2 (psi). Determine the volume at a pressure of 150 psi.

FUNCTIONS AND RELATIONS

CONCEPT OBJECTIVE: The student will identify examples of direct and joint variation.

EXAMPLES OF VARIATION

PROCEDURE: Given a list of common mathematical formulas, students can identify those examples of direct or joint variation. Interpreting the formulas using the vocabulary of variation, may also involve identification of the constant of proportionality. A sample list of formulas may include some which do not represent either direct or joint variation.

SAMPLE FORMULAS

$C = \pi d$

$P = 4s$

$P = a + b + c$

$A = \pi r^2$

$A = \frac{1}{2}bh$

$P = 2L + 2W$

$A = s^2$

$V = \pi r^2 h$

$F = \frac{9}{5}C + 32$

FUNCTIONS AND RELATIONS

CONCEPT OBJECTIVE: The student will use combined variation to solve problems.

COMBINED VARIATION

PROCEDURE: A given formula states $R = \frac{kL}{d^2}$, where R, the electrical resistance of a wire, varies directly as the length (L) and inversely as the square of the diameter of the wire (d^2). A specific application indicates that 25 meters of wire with a diameter of 1.5 mm has a resistance of 10 ohms. Determine the resistance of 25 meters of the same type of wire if the diameter is increased to 2mm.

CONCEPT OBJECTIVE: The student will determine if a relation is a function.

FUNCTIONS

PROCEDURE: *Algebra I, page 60.

FUNCTIONS AND RELATIONS

CONCEPT OBJECTIVE: The student will find the maximum value of a quadratic function.

CHARTER A BUS

PROCEDURE: A bus company will charter buses to groups of 30 people or more. For a group of exactly 30, the company charges $60 for each person. However, everyone's fare is reduced by 50 cents for each person in excess of 30. What size group will produce the largest revenue for the company, and what is this revenue?

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FUNCTIONS AND RELATIONS

CONCEPT OBJECTIVE: The student will practice identifying equations, graphs, and characteristics of circles, parabolas, ellipses, and hyperbolas.

C.H.E.A.P.

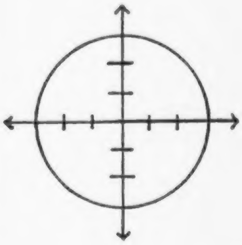
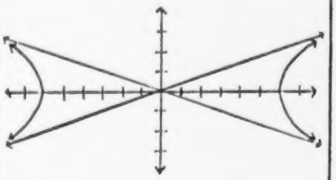
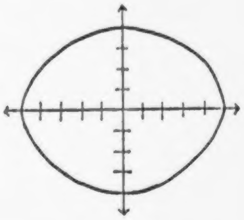
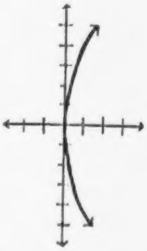
(CIRCLES, HYPERBOLAS, ELLIPSES, AND PARABOLAS)

PROCEDURE: Each group of 2-4 students needs a set of C.H.E.A.P. cards. After shuffling, the dealer deals 9 cards to each player. The rest of the deck is placed face down to form a stack. The top card is turned over and placed next to the stack to start the discard pile. Looking at the cards dealt as a hand, each player removes any sets of three matching cards. A matching set of three consists of an equation card, description card, and graph card describing the same figure.

After matching sets have been removed from each player's hand and placed on the table, play begins with the person on the left of the dealer. In a turn, a player may select the top card of the stack or the top card of the discard pile. The player must discard at the end of a turn. The first player to match all of the cards in his/her hand into sets of three is the winner. If all of the stack has been used before a winner is determined, the top card can be used to start a new discard pile, and the rest of the cards shuffled to form a new stack.

A deck of C.H.E.A.P. cards may consist of any number of sets of three matching cards. Each set of three cards must have equivalent equation, description and graph cards. There should be more than one set representing each of the four types of figures. Suggestions for cards of each type of figure are shown on the following page.

C.H.E.A.P. CARDS

CIRCLE	$x^2 + y^2 = 9$	RADIUS = 3 CENTER (0,0)	
HYPERBOLA	$\frac{x^2}{36} - \frac{y^2}{4} = 1$	ASYMPTOTES $y = \pm\frac{1}{3}x$	
ELLIPSE	$\frac{x^2}{25} + \frac{y^2}{16} = 1$	X-INTERCEPTS $(\pm 5,0)$ Y-INTERCEPTS $(0,\pm 4)$	
PARABOLA	$x = \frac{y^2}{16}$	FOCUS (4,0) DIRECTRIX $x = -4$	

FUNCTIONS AND RELATIONS

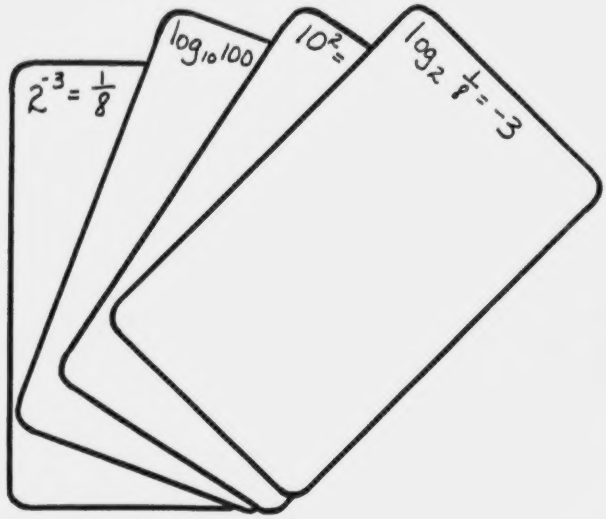
CONCEPT OBJECTIVE: The student will identify equivalent exponential and logarithmic notation.

LARRY LOGARITHM

PROCEDURE: Each group of 4-6 students will need a set of Larry Logarithm cards. After shuffling the cards, a dealer deals all cards face down to the players. Looking at the cards dealt as a hand, each player removes any matching pairs. A matching pair consists of an equation using exponential notation and an equivalent equation written using the logarithmic function.

The dealer begins play by drawing a card from the hand of the neighbor on the left. Play proceeds clockwise around the group, with each player drawing one card from the player on the left. Players continue placing matching pairs from their hands in front of them. The first player to match all of his/her cards is the winner. The other players continue the game until one person is left with the Larry Logarithm card. This player is the loser.

A deck of 33 cards may be created with 16 exponential notation cards, 16 logarithmic notation cards, and 1 Larry Logarithm card. Pairs of equivalent cards may take the form of $2^{-3} = \frac{1}{8}$ and $\log_2 \frac{1}{8} = -3$.



GRAPHING

CONCEPT OBJECTIVE: The student will construct a model for a three-dimensional graph.

TINKER WITH GRAPHING

PROCEDURE: Tinker toys can be used as a model for three-dimensional graphs. The x and y axes can be shown as the spokes on a wheel, using 4 sticks and a round connector. The spokes (axes) should be perpendicular to each other. The z-axis will be formed using a stick through the center of the wheel, perpendicular to the other two axes. Using a different color stick for each axis will make it easier for students to identify them. With this model, students can see the 8 octants formed by three axes. Three small pieces of plastic or cardboard can help the teacher to clearly illustrate a particular octant for discussion.

CONCEPT OBJECTIVE: The student will practice graphing line segments given a point and the slope.

AN UMBRELLA GRAPH

PROCEDURE: *Algebra I, page 61.

SERIES

CONCEPT OBJECTIVE: The student will apply the formulas for geometric series.

RUNNING FOR OFFICE

PROCEDURE: Jeffrey is running for student body president. He makes one campaign poster on the first day of the campaign. The next day he gets 3 campaign workers to make one poster each. Each of these 3 campaign workers finds 3 different students the third day to make posters, and so on. How many posters will be made on the seventh day of the campaign? How many posters will be made on the *n*th day? How many posters will Jeffrey have in all by the end of the seventh day of the campaign? How many posters will have been made in all by the end of the *n*th day of the campaign?

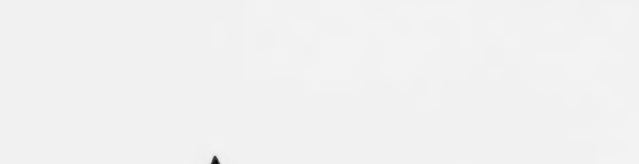
CONCEPT OBJECTIVE: The student will practice finding the sum and the *n*th term of a geometric series.

DOUBLE DO IT TO IT

PROCEDURE: Each student will need a sheet of $\frac{1}{4}$" graph paper and several different colored pencils or pens. Setting a time period of one minute, each student is to color one square of the graph paper. During the next minute, 2 squares are colored a different color. For the next minute, each student again colors twice as many squares as the previous minute, i.e., 4 squares. The process continues with the students coloring twice as many squares each minute as the previous time period until there are no more squares available to be colored.

Students can create a table setting up three columns, "Minute", "Number of Squares Colored", and "Total Squares Colored". Using this data, students can determine the expression for the *n*th term and the formula for the sum.

TRIGONOMETRY
STUDY OF ANGLES
FUNCTIONS AND IDENTITIES


Trigonometry

TRIGONOMETRY

Trigonometry provides the student with the study of periodic phenomena. The student also studies the relationship between sides and angles of triangles.

Functions and relations are used to determine measurements of triangles and circles.

Problems from the real world assist the student to become aware of the usefulness of trigonometry. Problem-solving skills may be developed through the use of written problems.

Proving identities reinforce the relationship between the basic trigonometric functions.

TOPIC	PAGE
Right Triangles	117
Vectors	120
Trigonometric Identities	122
Circular Functions	125
Graphing	133
Trigonometric Formulas	140

The contents of this chapter are organized by four strands coded with symbols as follows:

✏ - Activity

🎲 - Game

🌎 - Application

🔍 - Problem-solving Experience

Many activities are coded under two or more strands.

RIGHT TRIANGLES

CONCEPT OBJECTIVE: The student will practice using the properties of right triangles in trigonometry to solve problems.

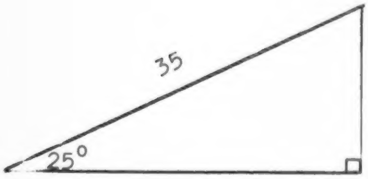
AIRPLANE FLIGHTS

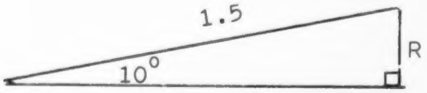
PROCEDURE: Review the definitions of sine, cosine and tangent as they relate to right triangles. Students can work problems such as the ones below.

PROBLEMS:

1. A radar station tracking an aircraft indicates the elevation angle to be 25° and the slant range to be 35 miles. Determine the altitude and horizontal range of the aircraft.

2. In one minute, an airplane climbing at a constant angle of 10° has flown a distance of 1.5 miles along its line of flight. Find the rate of climb of the airplane in miles per minute.





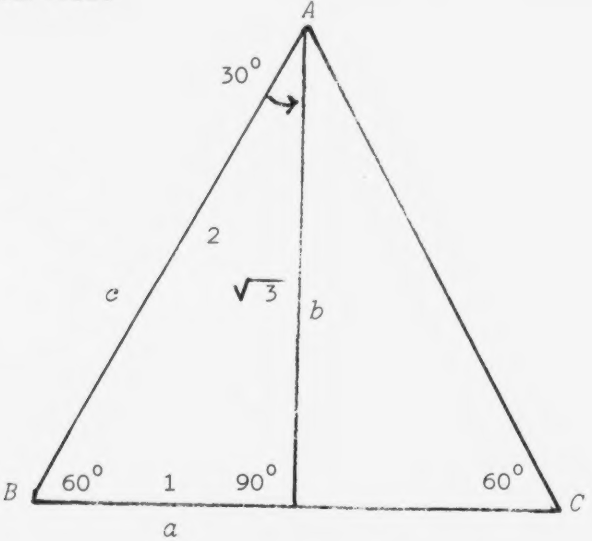
RIGHT TRIANGLES

CONCEPT OBJECTIVE: The student will derive values of special angles.

30-60-90

PROCEDURE: Begin with the drawing of an equilateral triangle having a measure of 2 units per side. Each angle of the triangle will measure 60°. Drop a perpendicular from a vertex to a side. This will bisect the angle at the vertex and a 90° angle is formed at the intersection of the bisector of the angle and the opposite leg. We now have formed a 30-60-90-degree angle with sides 1, 2 and $\sqrt{3}$. See drawing.

Using the definitions of the relations of the sides of a triangle with sine and cosine, the special angle values are derived.



$$\sin 60^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{b}{c} = \frac{\sqrt{3}}{2} = \cos 30^\circ = \frac{\text{adj}}{\text{hyp}}$$

$$\sin 30^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{a}{c} = \frac{1}{2} = \cos 60^\circ = \frac{\text{adj}}{\text{hyp}}$$

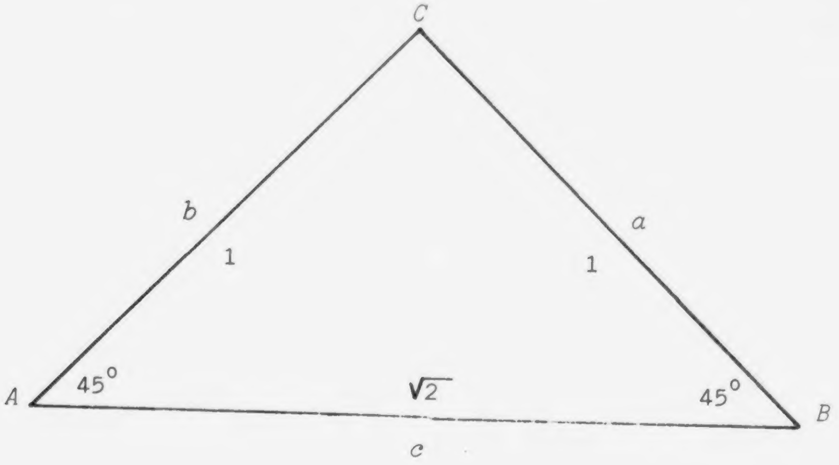
RIGHT TRIANGLES

CONCEPT OBJECTIVE: The student will derive values of special angles.

ISOSCELES TRIANGLES

PROCEDURE: Begin with an isosceles triangle with a 90° angle being formed at the vertex of the two equal legs. Find the measure of the vector connecting the end points. See the diagram below.

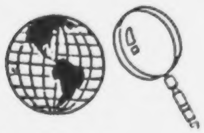
Use the definitions for the relationship of the sides of a triangle with sine and cosine to arrive at the values at 45°.



$$\frac{a}{c} = \sin 45^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}} = \cos 45^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{2}} = \frac{b}{c}$$

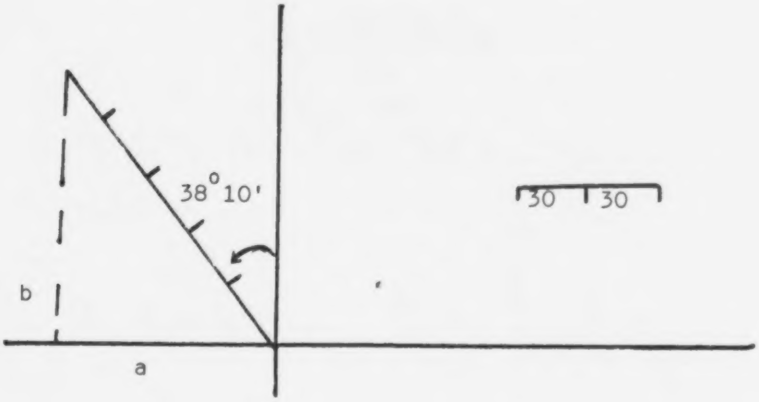
VECTORS

CONCEPT OBJECTIVE: The student will find the length of vectors by using trigonometric functions.

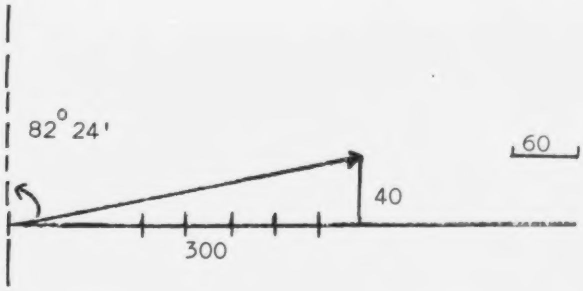


FLY WITH ME

PROCEDURE: An airplane flies 150 miles in the direction of N38°10'W. How far north and how far west of the starting point is the plane?



A plane is headed due east with airspeed 300 mph. If a wind of 40 mph from the south is blowing, find the groundspeed and track.

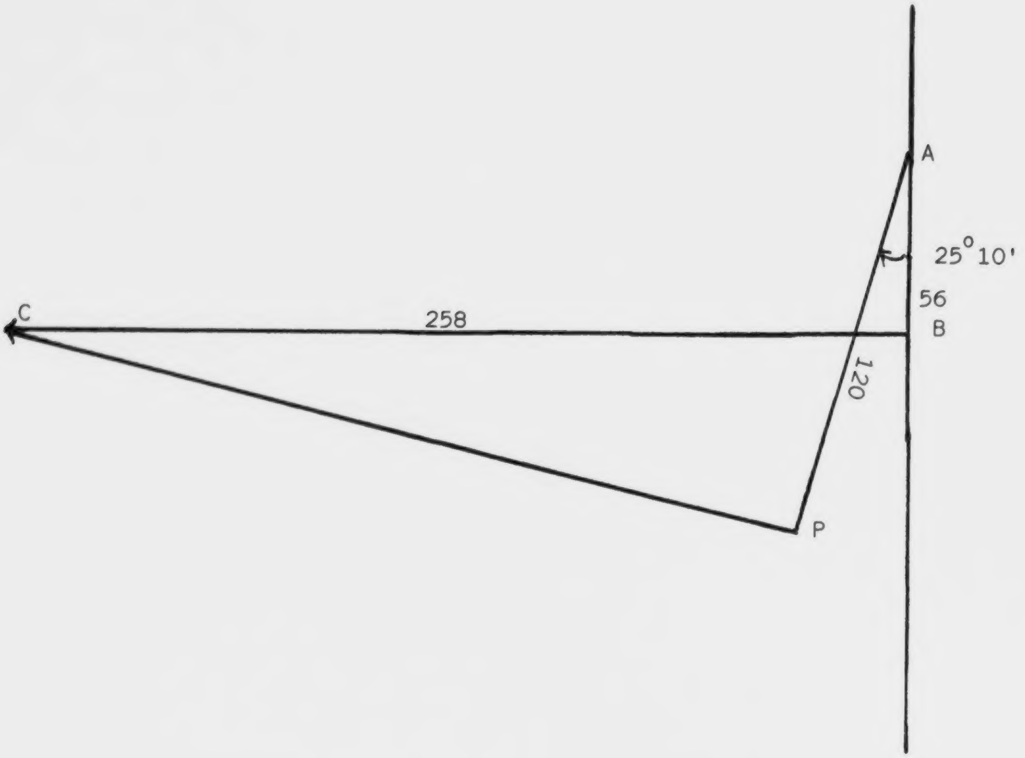


VECTORS

CONCEPT OBJECTIVE: The student will find the length of vectors by using trigonometric functions.

SAILING

PROCEDURE: A ship at A is to sail to C, 56 miles south and 258 miles west of A. After sailing S25°10'W for 120 miles to P, the ship is headed for C. Find the distance from P to C and the required course to reach C.



TRIGONOMETRIC IDENTITIES

CONCEPT OBJECTIVE: The student will develop strategies to solve and prove trigonometric identities.

SOS (STRATEGIES TO SOLVE)

PROCEDURE: After students have worked with some types of trigonometric identities, the following techniques may be introduced to solve or prove identities.

1. If one side involves only one function of an angle, express the other side in terms of this function.
2. If one side is factorable, factor it.
3. If one side has a monomial denominator, break up the fraction by applying the distributive law of division.
4. If one side is a sum and the other side a product, work with the sum.
5. When in doubt, express one or both sides in terms of sines and cosines.
6. Use techniques suggested by the terms in the identity. Avoid the use of radicals. Students must be cautioned against the cross multiply method on all identity proofs.

TRIGONOMETRIC IDENTITIES

CONCEPT OBJECTIVE: The student will use previously learned strategies to prove identities.

PROVING IDENTITIES

PROCEDURE: Review the 6 techniques to prove identities.

PROBLEMS: Verify the following identities.

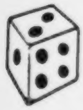
1. $\tan^2 x \csc^2 x \cot^2 x \sin^2 x = 1$

2. $\dfrac{\sin B}{\sin B + \cos B} = \dfrac{\sec B}{\sec B + \csc B}$

3. $\dfrac{\sin^3 B + \cos^3 B}{\sin B + \cos B} = 1 - \sin B \cos B$

TRIGONOMETRIC IDENTITIES

CONCEPT OBJECTIVE: The student will practice solving trigonometric identities.



TRIGOMINOES

PROCEDURE: Each pair of students needs a set of trigominoes and paper and pencil with which to keep score. Trigominoes are mixed up and placed face down on the table. Each player draws 13 trigominoes and the remaining one is turned face up in the middle to start the game. The rules of play and scoring are similar to dominoes. Each player tries to match one of his/her trigominoes with ones already played. Ends of trigominoes are matched end to end; however, a double is played perpendicular to the matching trigomino.

At the end of each play, points for that play are found by assigning the following point values to the ends of trigominoes that are unmatched: Double = 10 points; Single = 2 points; Fraction = 3 points; Graph = 5 points. On each turn, if one cannot match any of the trigominoes, he/she must pass to the other player. After all possible matches have been made, points are totaled to determine the winner.

A set of trigominoes may be found in the appendix.

CIRCULAR FUNCTIONS

CONCEPT OBJECTIVE: The student will convert degree measures to radian measures and radian measures to degree measures.

CONVERTIBLES

PROCEDURE: It is sometimes easier to do the arithmetic in trigonometry using radians rather than degrees, and sometimes it is the other way around. Explain to the students how the radian measure was derived and give them the formulas for the conversion from one to the other.

$m^{o}\measuredangle XOZ = \frac{180}{\pi} m^{R}\measuredangle XOZ \qquad m^{R}\measuredangle XOZ = \frac{\pi}{180} m^{o}\measuredangle XOZ$

Example 1. Change 120^{o} to radian measure.

$$m^{R} = \frac{\pi}{180} 120^{o} = \frac{2}{3}\pi$$

EXAMPLE 2. Change $\frac{4}{3}\pi$ to degree measure.

$$m^{o} = \frac{180}{\pi} (\frac{4}{3})\pi = 240^{o}$$

PROBLEMS:

Convert the following:

To Radians	To Degrees
144^{o}	$\frac{3}{4}\pi^{R}$
200^{o}	$\frac{2}{5}\pi^{R}$
45^{o}	$\frac{1}{3}\pi^{R}$

CIRCULAR FUNCTIONS

CONCEPT OBJECTIVE: The student will derive the values of special angles using radian measure.

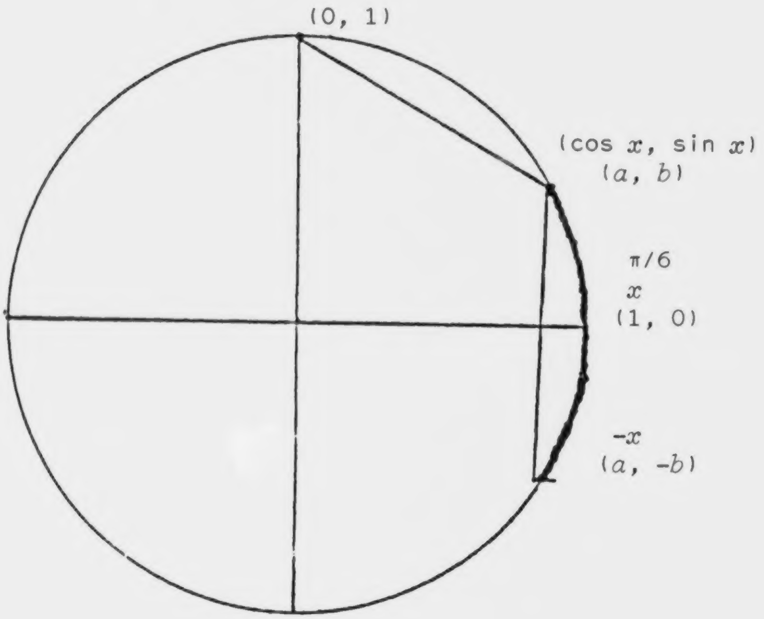
SPECIAL ANGLES

PROCEDURE: Begin with the review of trigonometric functions and the values of the six functions for the special angles.

PROBLEM:

Construct a unit circle as below. On the circumference of the circle, mark a distance x. In this particular case the distance will be $\pi/6$. At this point an ordered pair (a, b) or the ordered pair $(\cos x, \sin x)$ will be located.

At the point (1, 0) draw another distance x in the opposite direction along the circumference. This point is $(a, -b)$. The chord connecting the two points, (a, b) and $(a, -b)$, and the chord connecting (a, b) and (0, 1) are equal in length because both arcs along the circumference measure $\pi/3$ units.



SOLUTION:

To solve this problem use the distance formula. See page 217 for selected values of the six functions.

$$(a - 0)^2 + (b - 1)^2 = (a - a)^2 + (b - (-b))^2$$

$$a^2 + b^2 - 2b + 1 = 4b^2$$

$$-2b + 2 = 4b^2$$

$$0 = 4b^2 + 2b - 2$$

$$0 = (2b - 1)(b + 1)$$

$$2b = 1 = 0 \text{ or } b + 1 = 0$$

$$b = \frac{1}{2}$$

$$\text{or } b = -1 \text{ (reject) WHY?}$$

$$a^2 + b^2 = 1$$

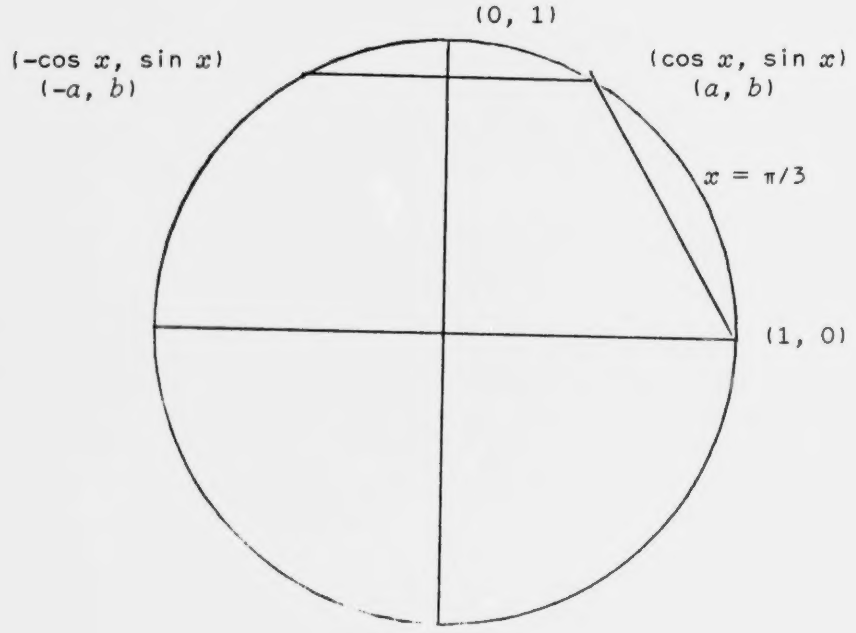
$$a^2 + \frac{1}{4} = 1$$

$$a^2 = \frac{3}{4}$$

$$a = \pm\frac{\sqrt{3}}{2}$$

$$\therefore \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2},\ \sin\frac{\pi}{6} = \frac{1}{2}$$

The same process can be used to find the values of cosine and sine of $\frac{\pi}{3}$.



Since the distance from (1,0) to (a,b) is equal to the distance from (a,b) to (-a,b), the arcs are equal. Solve using the distance formula.

SOLUTION:

$$(1 - a)^2 + (0 - b)^2 = (a - (-a)^2) + (b - b)^2$$

$$1 - 2a + a^2 + b^2 = 4a^2$$

$$2 - 2a = 4a^2$$

$$0 = 4a^2 + 2a - 2$$

$$(2a - 1)(a + 1) = 0$$

$$a = \tfrac{1}{2}$$

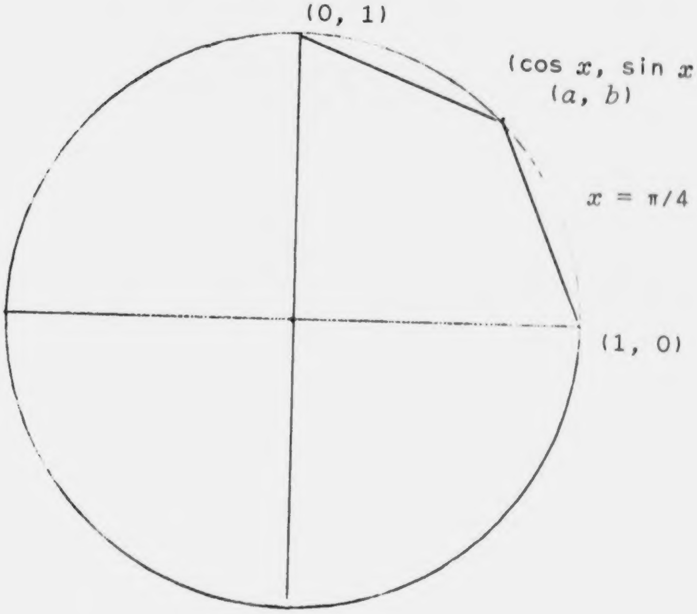
$$\text{or } a = -1 \text{ (reject)}$$

$$b = \frac{\sqrt{3}}{2}$$

$\therefore \cos \frac{\pi}{3} = \frac{1}{2}$, $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Because of symmetry, other values around the circle can be found.

The distance formula can be used to find the values of cosine and sine of $\frac{\pi}{4}$.



$$(1 - a)^2 + (0 - b)^2 = (a - 0)^2 + (b - 1)^2$$

$$1 - 2a + a^2 + b = a^2 + b^2 - 2b + 1$$

$$-2a = -2b$$

$$a = b$$

$$a^2 + b^2 = 1$$

$$a^2 + a^2 = 1$$

$$2a^2 = 1$$

$$a^2 = \tfrac{1}{2}$$

$$a = \frac{\sqrt{2}}{2}$$

$$b = \frac{\sqrt{2}}{2}$$

$\therefore$ cos $\frac{\pi}{4} = \frac{\sqrt{2}}{2}$, sin $\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

Other values can be found around the circle because of symmetry.

CIRCULAR FUNCTIONS

CONCEPT OBJECTIVE: The student will find areas of interior parts of circles by using central angles.

OPERATION ORANGLE

PROCEDURE: An orange is cut in a manner so that each section is shown. If the orange has a diameter of 12 cm and has 11 sections, what is the area of each section?

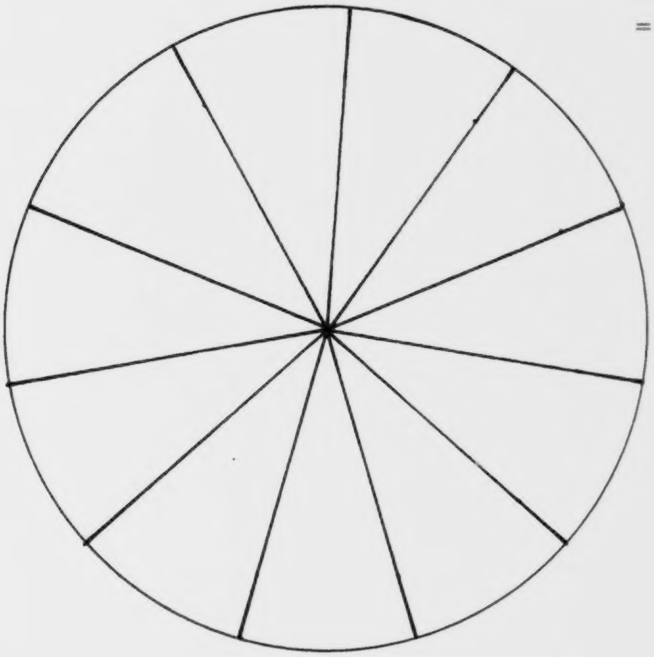
The student will need to generate a general formula for dividing the circle into equal sections created by central angles. This can be done as follows:

Define this as a portion of the interior of a circle cut by a central angle. Assume the circle has radius r. The sum of the measure of all the angles is 2π radians. If the central angle has a measure of θ radians, the section makes up a fraction of a complete circle $\theta/2\pi$. The area of a circle is πr^2. Hence the area of the section can be given as $\theta/2\pi$ times πr^2. When this reduced, it is $\frac{1}{2}\theta r^2$.

Students can use this information to complete the problem. Each section divides the circle into approximately $32^\circ 40'$, or each central angle measures $32^\circ 40'$. Using the formula:

$\frac{1}{2}$ times the value of $32^\circ 40'$ in radians times $6^2 = \frac{1}{2}(.5701)(36)$

$= 10.26$ sq. cm.



CIRCULAR FUNCTIONS

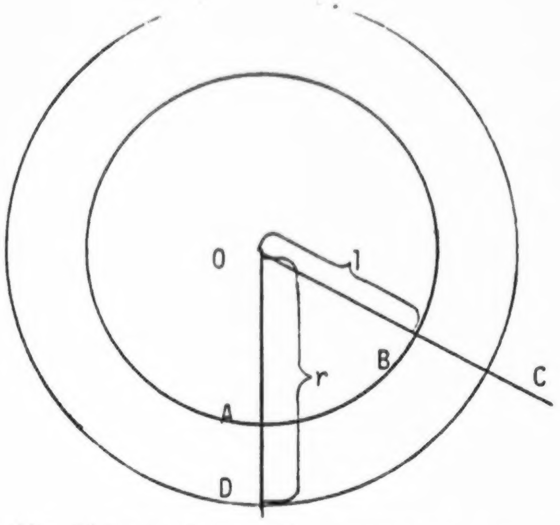
CONCEPT OBJECTIVE: The student will measure the length of an arc using central angles and radian measure.

ORANGE AID

PROCEDURE: Discuss with the students the measure of a chord, and the method to determine the measure of the length of an arc.

To derive a formula to measure the length of arcs, the fact that the length of an arc is proportional to the measure of its central angle is needed.

Using two concentric circles, one a unit circle, the second with radius 4, the corresponding lengths of arc on the circles subtending a given central angle are proportional to the circumferences of the circles.



Using the figure above, the length of arc DC is to $2\pi r$ as the length of arc AB is to 2π. The measure of angle BOA is p radians, then length of arc DC is pr.

Going back to Operation Orange in the preceding problem, the length of each arc of each section is .5701 × 6 = 3.4 approximately.

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CIRCULAR FUNCTIONS

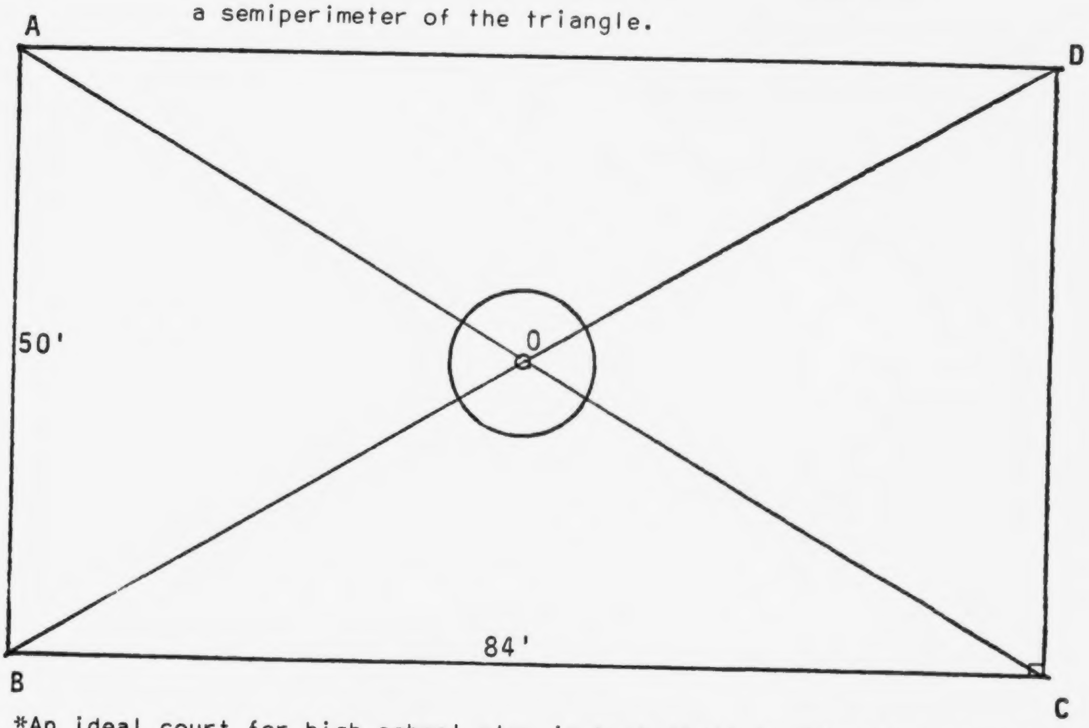
CONCEPT OBJECTIVE: The student will find the central angle by using radian measure.

BOUNCE

PROCEDURE: A discussion can be held about the local basketball court. Information from the National Federation Basketball Rule Book concerning the radius of each of the circles at mid-court and the size of the ideal court, can be shared with the students.*

Using the diagram below, determine the measure in degrees of the angle between *OB* and *OC*; of *OC* and *OD*. What is the length of the arc between these two line segments? What is the area of triangle *BOC* and triangle *COD*? (Try Heron's Formula** for one area.)

$K = \sqrt{s(s - a)(s - b)(s - c)}$ where s is $\frac{a + b + c}{2}$, a semiperimeter of the triangle.



*An ideal court for high school play in basketball is 50 x 84 feet. The two circles at midcourt have radii of 2 and 6 feet respectively.

**Also known as Hero's Formula.

GRAPHING

CONCEPT OBJECTIVE: The student will sketch the graph of a function in the form of $y = a \sin bx$ or $y = a \cos bx$, where $b>0$.

GRAPHING SINE AND COSINE FUNCTIONS

PROCEDURE: The following are steps to sketch the graph of $y = a \sin bx$ or $y = a \cos bx$.

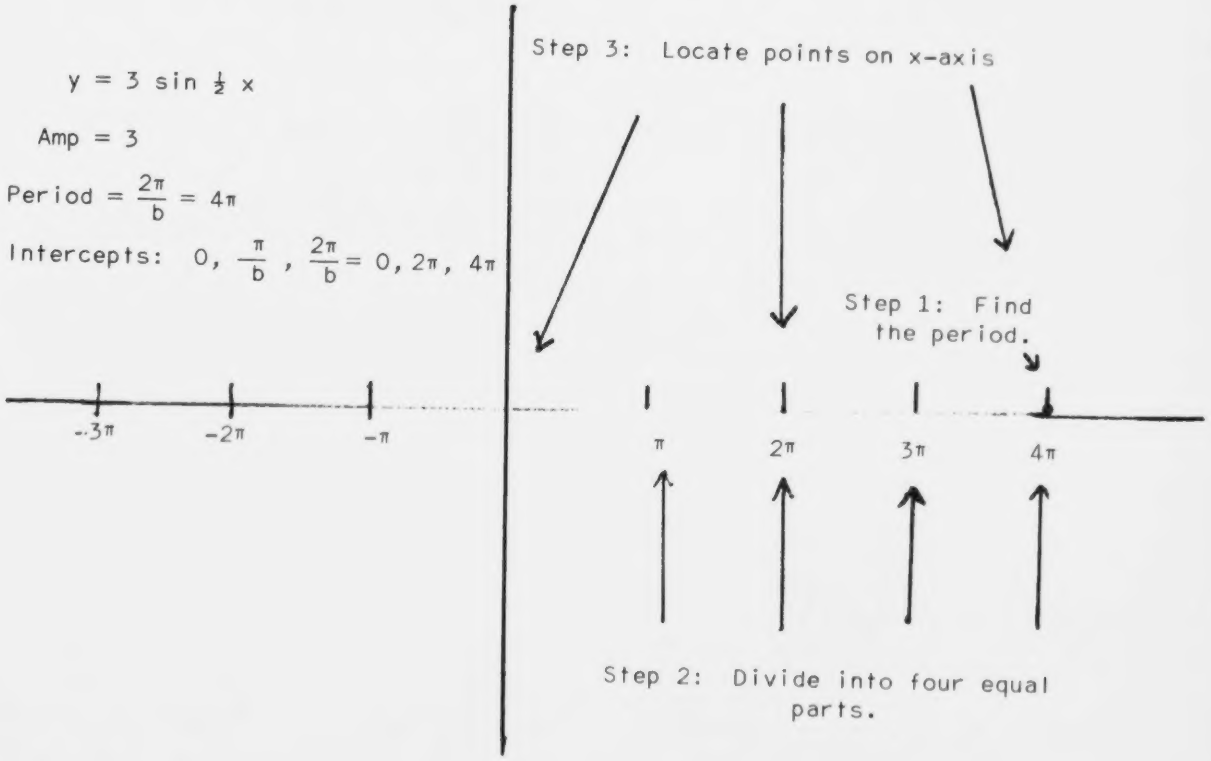
1. Find the period, $2\pi/b$. Start at 0 on the x-axis and lay off a distance of $2\pi/b$.
2. Divide the interval from 0 to $2\pi/b$ into four equal parts.
3. Locate the points where the graph crosses the x-axis. For the function, $y = a \sin bx$, the graph crosses the x-axis at 0, π/b, and $2\pi/b$. These points will be the beginning, middle, and end of the interval. For the function, $y = a \cos bx$, the graph crosses the x-axis at $\pi/2b$ and $3\pi/2b$. These points will be locations at one-fourth and three-fourths distances across the interval.
4. Locate the points where the graph reaches its maximum and minimum values.
   For the function, $y = a \sin bx$, the maximum will be $\pi/2b$ when $a>0$. When $a<0$, the maximum will be $3\pi/2b$.
   For the function, $y = a \cos bx$, the maximum will be 0 when $a>0$. When $a<0$, the maximum will be π/b.
5. Sketch the graph.
6. Draw additional periods of the graph to the right and left as needed.

$y = 3 \sin \frac{1}{2} x$

Amp = 3

$\text{Period} = \frac{2\pi}{b} = 4\pi$

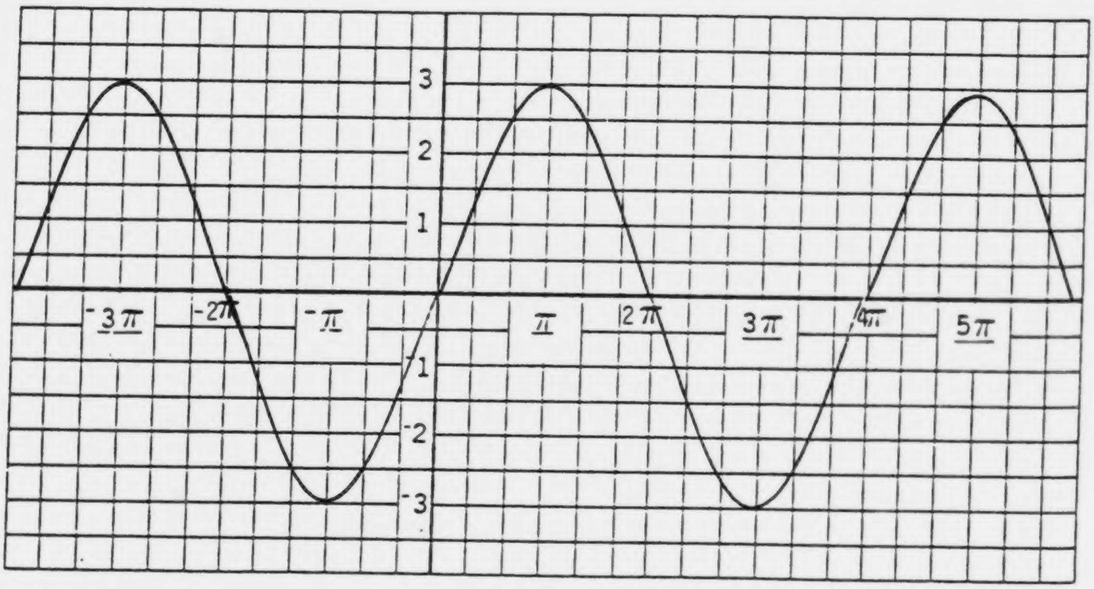
$\text{Intercepts: } 0, \frac{\pi}{b}, \frac{2\pi}{b} = 0, 2\pi, 4\pi$


Step 3: Locate points on x-axis

Step 1: Find the period.

Step 2: Divide into four equal parts.

$y = 3 \sin \frac{1}{2}x$



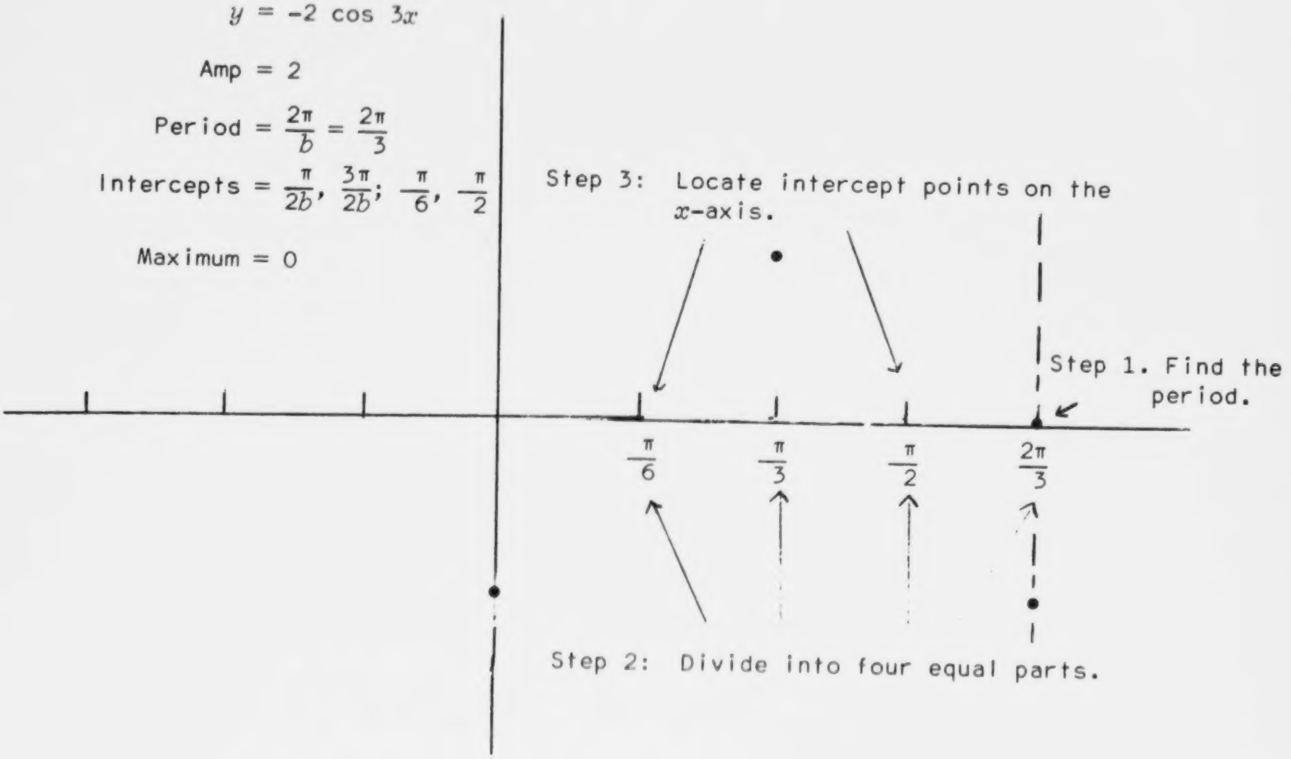
$y = -2 \cos 3x$

Amp = 2

Period = $\frac{2\pi}{b} = \frac{2\pi}{3}$

Intercepts = $\frac{\pi}{2b}, \frac{3\pi}{2b}; \frac{\pi}{6}, \frac{\pi}{2}$

Maximum = 0

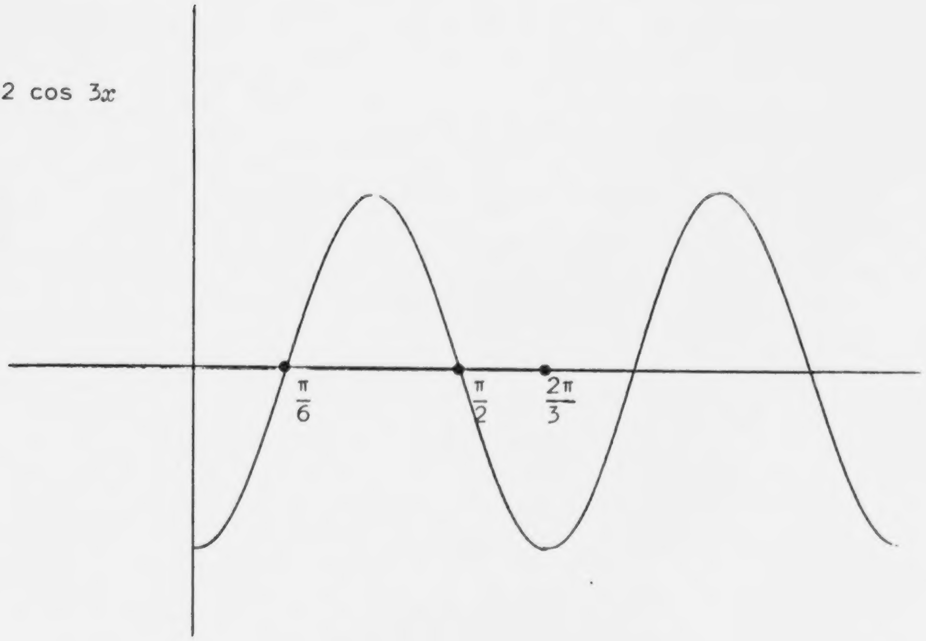

$\frac{\pi}{6}$
$\frac{\pi}{3}$
$\frac{\pi}{2}$
$\frac{2\pi}{3}$

Step 3: Locate intercept points on the x-axis.

Step 1. Find the period.

Step 2: Divide into four equal parts.

$y = -2 \cos 3x$



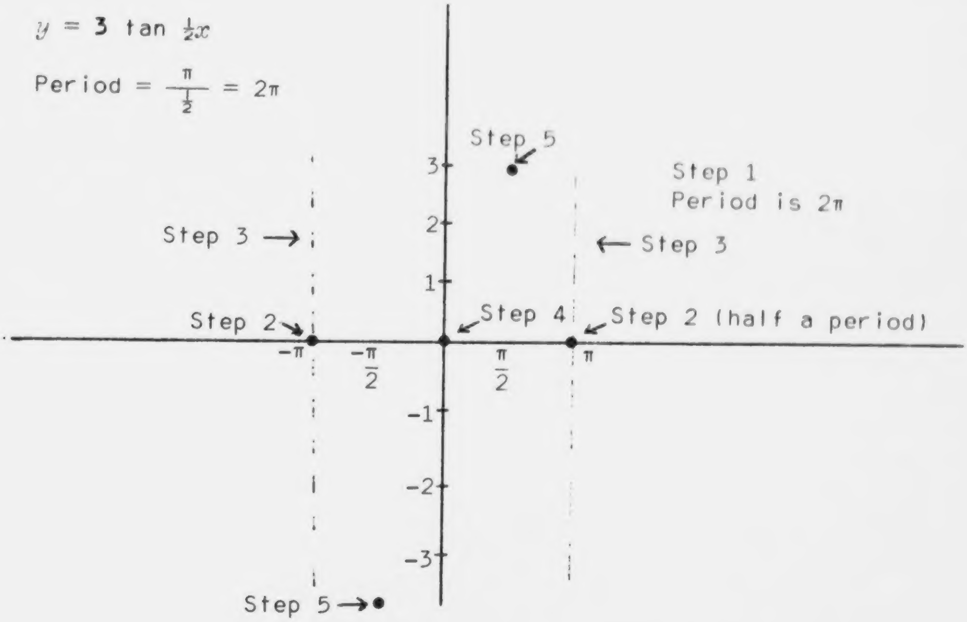
GRAPHING

CONCEPT OBJECTIVE: The student will sketch the graph of equations containing tangent and cotangent functions.

GRAPHING TANGENT AND COTANGENT FUNCTIONS

PROCEDURE: The following are steps to sketch the graph of $y = a \tan bx$ or $y = a \cos bx$.

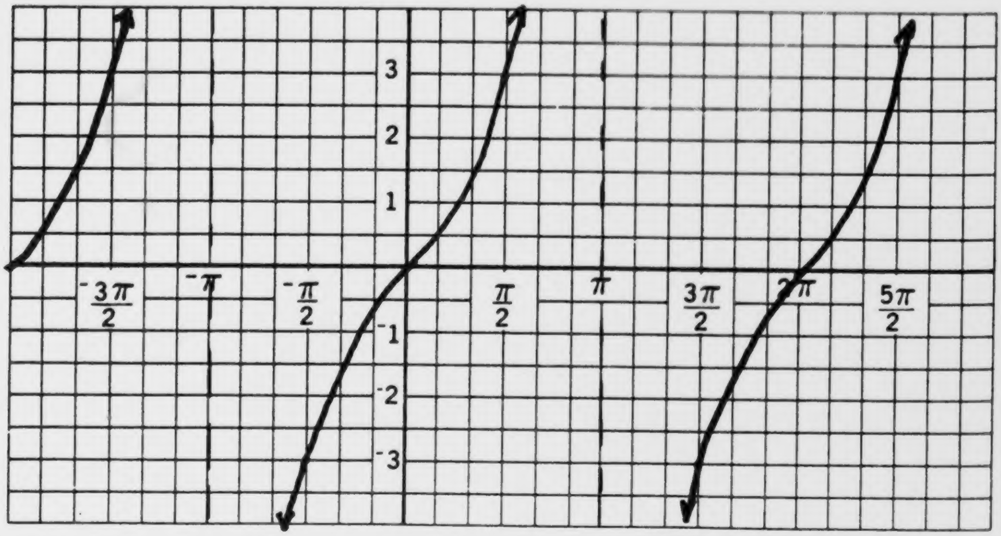
1. Find the period, π/b.
2. Start at 0 on the x-axis and lay off two intervals, each with length half the period. One interval goes to the left and the other goes to the right of 0.
3. Draw the asymptotes as vertical dotted lines at the end-points of the interval created in Step 2 for tangent and along the y-axis for cotangent.
4. For tangent, locate a point at (0,0). For cotangent, locate points on the x-axis at each end of the interval of step 2.
5. Sketch the graph, finding additional points as needed.
6. Draw additional periods both to the right and left as needed.



$y = 3 \tan \frac{1}{2}x$

$\text{Period} = \frac{\pi}{\frac{1}{2}} = 2\pi$

$y = 3 \tan \frac{1}{2}x$



GRAPHING

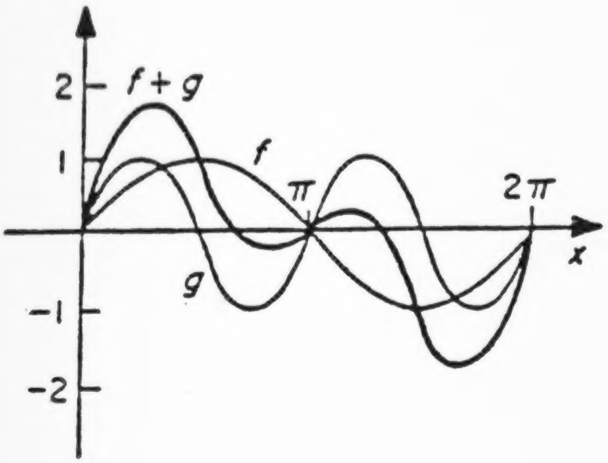
CONCEPT OBJECTIVE: The student will graph trigonometric functions.

COMBINATION OF FUNCTIONS

PROCEDURE: Students can practice graphing various trigonometric functions.

The combination of the functions can have some startling results. Students can graph the following functions on the same set of coordinates. Using a table, ruler or compass, the sum can be found and the results plotted.

$f(x) = \sin x;\ g(x) = \sin 2x;\ 0 \leq x \leq 2\pi$



The solid line is the result of adding the two functions.

Students can try these combinations.

PROBLEMS:

1. $y = \sin x + \frac{1}{2} \cos 2x$

2. $y = \sin x + \sin 2x + \sin 3x$

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GRAPHING

CONCEPT OBJECTIVE: The student will practice finding locations on a polar coordinate system.

POLAR TIC-TAC-TOE

PROCEDURE: Each pair of players needs a sheet showing a polar coordinate system using circles of radius 1, 2, 3, and 4 and radial lines every 30 degrees. The origin is omitted as a playing point because its position is unfairly advantageous.

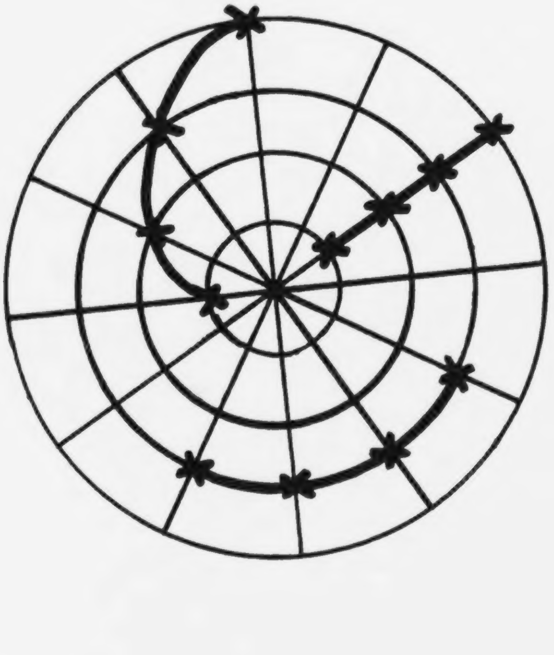
Players alternate naming a location for placement of their X's and O's at the intersections of the lines and circles. Locations must be stated correctly in the form (r,θ) and must match the placement of the player's mark. If players incorrectly identify a location, they lose their turn. Radian notation may be used instead of degrees.

Players may win by placing four of their marks in a row in one of three ways:

1) along a radial line;

2) along a circle;

3) along a diagonal or spiral.



TRIGONOMETRIC FORMULAS

CONCEPT OBJECTIVE: Given the measure of three parts of a triangle, at least one being a side, the student will find the other parts of the triangle using the law of cosines and the law of sines.

LAW OF COSINES

PROCEDURE: Students are reminded that the law of cosines will be used if three sides of the triangle are known, or if the measures of two sides and the angle determined by them are given. Otherwise the law of sines must be used.

Given $a = 6$, $b = 8$, $\gamma = 60$

Find c, α, β

$c^2 = a^2 + b^2 - 2ab \cos \gamma$

$c^2 = 36 + 64 - 2(6)(8)\cos 60^\circ$

$c^2 = 36 + 64 - 2(6)(8)(\frac{1}{2})$

$c^2 = 100 - 48 = 52$

$c = \sqrt{52} \doteq 7.2$

$b^2 = a^2 + c^2 - 2ac \cos \beta$

$64 = 36 + 52 - 2(6)(7.2)\cos \beta$

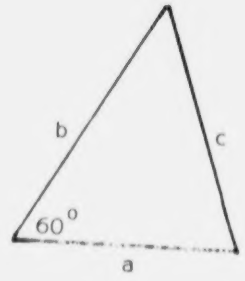
$64 = 88 - 2(6)(7.2)\cos \beta$

$\frac{64 - 88}{-12\ (7.2)} = \cos \beta$

$.2800 = \cos \beta$

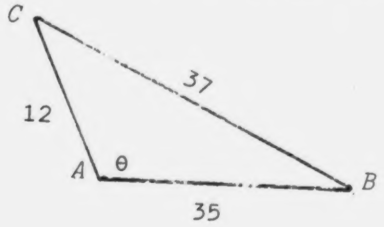
$\beta = 74$

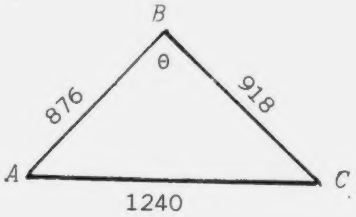
$\alpha = 180 - 74 - 60 = 46$



PROBLEMS:

Use the law of cosines to find θ in the following figures. Round off to the nearest 10'.





TRIGONOMETRIC FORMULAS

CONCEPT OBJECTIVE: The student will practice using the law of cosines and the law of sines in practical applications.

DEADMAN

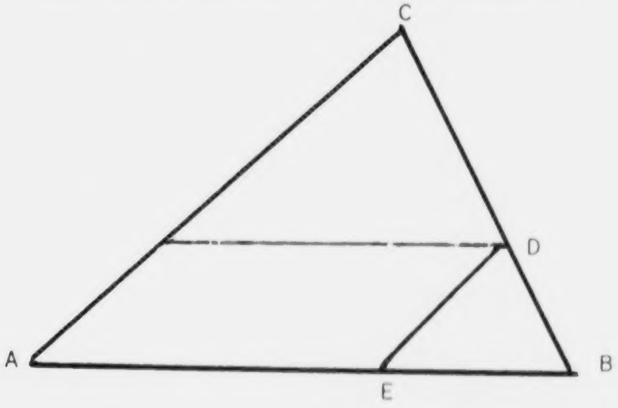
PROCEDURE: Review the law of cosines and the law of sines. Make a drawing that will lead the students into the use of their knowledge of sums of angles, sums of measures of angles in triangles and the use of the law of cosines and the law of sines.

PROBLEM:

A triangular field has one side parallel to a highway and is 3/8 of a mile long. The other two sides intersect this first side at points A and B with interior angles of 43° and 64° respectively. At point C there is a "deadman"*. There is another fence 3/32 of a mile along BC parallel to the first fence. Another fence also begins at this point (d) and intersects AB 1/4 of a mile from A called E.

1. What is the length of DE?

2. How far is it from A to the "deadman"?



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*In early fence building, large rocks or logs were placed in holes in order to aid in stretching the fence. These were called "deadmen".

TRIGONOMETRIC FORMULAS

CONCEPT OBJECTIVE: The student will solve problems using the law of sines.

FIND THE RADIO

PROCEDURE: A few years ago a student in southwestern Oklahoma constructed a radio transmitter. The transmissions interfered with conversations between the airbase and aircraft in the vicinity. Base personnel were able to locate the transmitter by means of triangulation.

PROBLEMS:

1. A listening station was located at the control tower on the airbase. A second listening station was placed 5 miles north. From the control tower it was determined that the sound was coming from a bearing of 26°. From the second station the sound came from 97° measured from the north. How far was it to the transmitter from each listening post?

2. Suppose that from the control tower a listening post is established 18 km due north. The control tower post noted that the transmissions were coming from N 35° E. Information received also stated that the transmitter was located 14 km from post 2. In constructing a diagram on coordinate graph paper it was revealed the transmitter was located only 5.30 km from the control tower. However, this was not the case. Explain what happened.

TRIGONOMETRIC FORMULAS

CONCEPT OBJECTIVE: The student will use a formula containing a trigonometric function to solve problems.

MACH 2

PROCEDURE: After the discussion of the formulas of the half angles, the following is one use of them.

Sound waves coming from a plane flying faster than the speed of sound form a cone. The cone intersects with the earth in the form of a hyperbola. A sonic boom is heard at the point of the intersection of this hyperbola and the earth.

The formula for finding either the speed of the plane or the angle at the vertex of the cone is

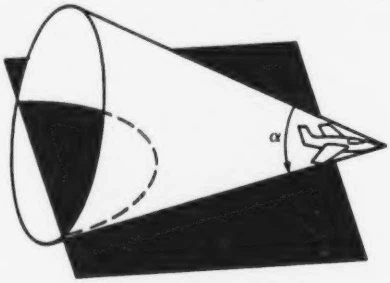
$$\sin \frac{\alpha}{2} = \frac{1}{m}$$

where m is the mach number* of the plane and alpha is the measure of the angle. Assume $m > 1$.

PROBLEMS:

1. Find the speed of a jet bomber if the measure of the angle is 30°; 60°; 45°.
2. Find the measure of the angle of the cone if the speed of an aircraft is $m = 2$; $m = \frac{3}{2}$; $m = \frac{5}{2}$.

*Mach number is the ratio of the speed of the plane and the speed of sound.



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ANALYSIS

ADVANCED MATH CONCEPTS

STUDYING, THINKING, AND LEARNING

ANALYSIS

Analysis will prepare the student to study advanced mathematics through the examiniation of real and complex numbers, polynomials, and analytic geometry.

The student will be able to recognize manipulative techniques to facilitate the application of these techniques in the solution of problems.

The breadth and depth of the applications of mathematics will be recognized by the student.

TOPIC	PAGE
Sequences and Series	149
Graphing	160
Conic Sections	167
Complex Numbers	171
Polynomial Equations	173
Applications of the Derivative	175

The contents of this chapter are organized by four strands coded with symbols as follows:

✏ - Activity

🌎 - Application

🎲 - Game

🔍 - Problem-solving Experience

Many activities are coded under two or more strands.

SEQUENCES AND SERIES

CONCEPT OBJECTIVE: The student will find the *nth* term, common difference, and the first term given a progression with any part missing.

ARITHMETIC PROGRESSION

PROCEDURE: Review the formula for arithmetic progressions $t_n = a + (n - 1)d$.

Problems:

1. Find the 12th term of an arithmetic progression whose first term is -8, with a common difference of 4.
2. Find the common difference of an arithmetic progression whose first term is 12 and $t_8 = 124$.
3. Find the 5 arithmetic means between -2 and 22.
4. Find 3 numbers of an arithmetic progression whose sum is 33 and whose product is 627.

CONCEPT OBJECTIVE: The student will practice finding terms of an arithmetic progression.

POST

PROCEDURE: Review the formula for finding the terms of an arithmetic progression.

PROBLEM: (See problem Enclosures page 176)

A rancher has 25 heavy-duty posts to place at equal distances along the front of a parcel of land which measures 800 feet. Determine the points, in feet, these posts should be placed.

SEQUENCES AND SERIES

CONCEPT OBJECTIVE: The student will work with progressions to derive common differences.

USING COMMON DIFFERENCES

PROCEDURE: Within discussions of arithmetic progressions, have students remember that to get from one member of a sequence to another is to add the common difference. In turn, if you have a sequence, subtract from a term the preceding term to find the difference.

For example, to find the common difference in the sequence $x + y$, $2x + 3y$, $3x + 5y$, subtract $x + y$ from $2x + 3y$ and $2x + 3y$ from $3x + 5y$. The difference is the same, $x + 2y$.

PROBLEM:

Find the value of x if the sequence $x - 1$, $2x + 3$, and $4x - 5$, in this order forms an arithmetic progression.

SOLUTION:

Subtract as in the example and set the differences equal to each other:

$$\begin{aligned} 2x + 3 - (x - 1) &= 4x - 5 - (2x + 3) \\ 2x + 3 - x + 1 &= 4x - 5 - 2x - 3 \\ x + 4 &= 2x - 8 \\ 12 &= x \end{aligned}$$

Thus the sequence is 11, 27, 43

SEQUENCES AND SERIES

CONCEPT OBJECTIVE: The student will find the *nth* term of a geometric progression, or any part of the given progression not given.

GEOMETRIC PROGRESSION

PROCEDURE: Review the formula for finding the *nth* term of a geometric progression, $t_n = ar^{n-1}$.

PROBLEMS:

1. Find the 7th term of the geometric progression having $a = \frac{1}{2}$ and $r = 2$.
2. Insert three real geometric means between 2 and 32.
3. Find a if $t_n = 96$, $r = -2$ and $n = 7$.
4. Find r if $a = 2$, $t_6 = 486$.

SEQUENCES AND SERIES

CONCEPT OBJECTIVE: The student will practice finding ratios and geometric sequences.

USING COMMON RATIOS

PROCEDURE: A common ratio between terms of a geometric progression can be found by dividing each term of the sequence by the preceding term.

In general terms, $t_n = ar^{n-1}$ or $t_n = t_{n-1}r$. Thus, $r = \frac{t_n}{t_{n-1}}$

An example of a geometric progression is 12, −3, 3/4, −3/16. Divide −3 by 12; 3/4 by −3, etc., and the ratio is −1/4.

PROBLEM:

Find the value of x in the following sequence so that a geometric progression will result.

$10 - 3x,\ x - 4,\ 2x - 5.$

SOLUTION:

The ratio between each pair must be the same. Set up a proportion to show this pair of ratios.

$$\frac{x - 4}{10 - 3x} = \frac{2x - 5}{x - 4}$$

$$(x - 4)(x - 4) = (10 - 3x)(2x - 5)$$

$$x^2 - 8x + 16 = -6x^2 + 35x - 50$$

$$7x^2 - 43x + 66 = 0$$

$$x = 3,\ 22/7$$

When $x = 3$, the geometric progression is 1, −1, 1, with a common ratio of −1.

When $x = 22/7$, the geometric progression is 4/7, −6/7, 9/7, with a common ratio of −3/2.

SEQUENCES AND SERIES

CONCEPT OBJECTIVE: The student will find the sum, number of terms or a member of the series given information concerning the series.

ARITHMETIC SERIES

PROCEDURE: Review the formula for finding the sum of an arithmetic series $S_n = \frac{n}{2}[2a + (n - 1)d]$ or $S_n = \frac{n}{2}(a + l)$ where l is the last term or t_n.

PROBLEMS:

1. Find the sum of the first 14 terms of the series $-3 + 1 + 5 + 9 + \ldots\ldots$
2. The sum of 22 terms of an arithmetic series is 161. If the first term is 14, find d.
3. After inserting k arithmetic means between 4 and 44, the sum of the resulting progression is 240. Find k.

SEQUENCES AND SERIES

CONCEPT OBJECTIVE: The student will practice using the formula for finding the sum of a geometric series.

DOLLARS & SENSE

PROCEDURE: Review the forumla for finding the sum of a finite geometric series.

PROBLEMS:

During the summer vacation from school, you find a job that you enjoy. To amass money to go to college, you decide to put into the bank 10 cents the first day, 20 cents the second day, 40 cents the third day and 80 cents the fourth day, and so on.

A. How much money would you have deposited during the days 1-15 inclusive?

B. How much money would you have deposited for the entire month of June?

C. How much will be deposited on the 30th day?

SOLUTIONS:

A. $a = .10$

$r = 2$

$$S_{15} = \frac{(1 - r^{15})}{1 - r} = \frac{.1(1 - 2^{15})}{1 - 2} = .1(2^{15} - 1)$$

$$= .1(32{,}768 - 1)$$

$$= .1(32{,}767)$$

$$= \$3{,}276.70$$

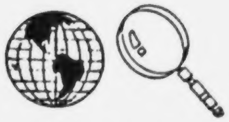
B. $2^{30} = 2^{15} \cdot 2^{15} = (32,768)(32,768) = 1,073,741,824.$

$.1(1,073,741,824 - 1) = .1(1,073,741,823) = \$107,374,182.30$

C. $2^{29} = 2^{15} \cdot 2^{14} = (32,768)(16,384) = \$536,870,912.$

$t_{30} = ar^{n-1} = .1(r^{29}).1(2^{29}) = \$53,687,091.20.$

CONCEPT OBJECTIVE: The student will use the formula to find the sum of a finite geometric series or any part of the series after receiving information concerning the series.



FINITE GEOMETRIC SERIES

PROCEDURE: Review the formula for finding the sum of a finite geometric series:

$$S_n = \frac{a - ar^n}{1 - r}$$

PROBLEMS:

1. The sum of 6 terms of a geometric series is -105. If $r = 2$, find a.

2. The sum of three numbers in an arithmetic progression is 24. If the first is decreased by 1 and the second is decreased by 2, the numbers are in geometric progression. Find the arithmetic progression.

SEQUENCES AND SERIES

CONCEPT OBJECTIVE: The student will practice finding terms of a geometric series.

FIRST, LAST, AND ALWAYS

PROCEDURE: Review the formula for finding the sum of a geometric series.

PROBLEMS:

1. An infinite geometric series has the sum 48. If the sum of the first two terms is 21, find a and r.
2. An infinite geometric series has 3 as its sum. If the second term is 5/12, find a and r.

SOLUTIONS:

1.

$$S = 48$$

$$a + ar = 21$$

$$a = 21/(1 + r)$$

$$48 = \frac{21/(1 + r)}{1 - r}$$

$$48 - 48r^2 = 21$$

$$-48r^2 = -27$$

$$r^2 = 9/16$$

$$r = \pm 3/4$$

If $r = 3/4$, $a = 12$

If $r = -3/4$, $a = 84$

2.

$$S = 3$$

$$ar = 5/12$$

$$a = (5/12)r$$

$$3 = \frac{(5/12)r}{1-r}$$

$$3 - 3r = 5/12$$

$$36r - 36r^2 = 5$$

$$36r^2 - 36r + 5 = 0$$

$$(6r - 5)(6r - 1) = 0$$

$$r = 5/6,\ 1/6$$

If $r = 5/6$, $a = 1/2$

If $r = 1/6$, $a = 5/2$

CONCEPT OBJECTIVE: The student will find the sum of a geometric series, or any missing part of the series given proper information.

INFINITE GEOMETRIC SERIES

PROCEDURE: Review the forumla of the sum of an infinite geometric series and how it was derived.

$$S = \frac{a}{1 - r}$$

PROBLEMS:

1. Find the sum of the series 24, 12, 6, . . .
2. Find the sum of the series 18 - 12 + 8 . . .

SEQUENCES AND SERIES

CONCEPT OBJECTIVE: The student will determine when a series is convergent or divergent.

COMPARISONS

PROCEDURE: Normally to test for convergency, each term of the series is compared with each corresponding term of a series that is known to converge. If each term is less than the compared term, (some authors call this bounding) then the series is convergent. If each term of a given series is equal to or greater than the corresponding term of a divergent (known) series, the given series is divergent. If, in a series of positive terms, the test ratio which is given below, approaches a limit r as n approaches infinity, the series is convergent if r is less than 1 and is divergent if r is greater than 1. For $r = 1$, the ratio test will fail to indicate either divergency or convergency.

$$r_n = \frac{s_{n+1}}{s_n}$$

EXAMPLE: $\frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{1}{4} + \ldots\ldots$

or

$$\frac{1}{2} + \frac{1}{2} + \frac{3}{2} + \frac{4}{2} + \ldots\ldots$$

$$s_n = \frac{n}{2^n} \text{ and } s_{n+1} = \frac{n+1}{2^{n+1}};\ r_n = \frac{n+1}{2^{n+1}} \cdot \frac{2^n}{n} = \frac{n+1}{2n}$$

Then $r = \lim_{n\to\infty} r_n = \frac{n+1}{2n} = \lim_{n\to\infty} \frac{1 + 1/n}{2} = 1/2$ which is less than 1. Therefore, the series is convergent.

EXAMPLE: $\dfrac{2}{1^2 + 1} + \dfrac{2^3}{2^2 + 3} + \dfrac{2^5}{3^2 + 3} + \cdots\cdots\cdots$

$$s_n = \frac{2^{2n-1}}{n^2 + n}, \quad s_{n+1} = \frac{2^{2n+1}}{(n+1)^2+(n+1)};$$

$$r_n = \frac{2^{2n+1}}{(n+1)^2+(n+1)} \cdot \frac{n(n+1)}{2^{2n-1}} = \frac{4n}{n + 2}$$

$\therefore\ r = \lim\limits_{n\to\infty} \dfrac{4}{1 + 2/n} = 4>1$ and is divergent.

GRAPHING

CONCEPT OBJECTIVE: The student will graph or sketch the equations of the conic section.

CIRCLES

PROCEDURE: Solve the equation for *x* or *y* and use normal procedures for finding values to plot.

PROBLEMS: Graph the following.

A. $x^2 + y^2 = 36$

B. $x^2 + (y - 2)^2 = 36$

C. $(x - 2)^2 + y^2 = 36$

D. $(x - 2)^2 + (y - 2)^2 = 36$

Graph any two of the above on the same set of axes. What are the points of intersection?

This should naturally lead into the algebraic solution of quadratic-quadratic equations.

GRAPHING

CONCEPT OBJECTIVE: The student will graph the locus of a parabola by plotting points.

PARABOLAS

PROCEDURE: Solve the equation for either x or y, depending on which variable is the dependent variable. Assign values for the independent variable to obtain the value of the function or the value of the dependent variable.

PROBLEMS: Graph the following.

A. $y = x^2$

B. $y = x^2 - 2$

C. $y = (x - 2)^2$

D. $y = x^2 - 4x + 6$

Questions to ask:

1. What effect does each constant have on the graph?
2. Does the placement of parenthesis have any effect on the graph?

GRAPHING

CONCEPT OBJECTIVE: The student will graph the locus of an equation by plotting points.

ELLIPSES

PROCEDURE: Solve the equation for either variable. Estimate values for the irrational numbers obtained. Find the center of the ellipse and the vertex along each axis (both major and minor axes).

PROBLEMS: Graph the following.

A. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

B. $\frac{x^2}{36} + \frac{y^2}{9} = 1$

C. $\frac{x^2}{20} + \frac{y^2}{25} = 1$

D. $\frac{y^2}{49} + \frac{x^2}{9} = 1$

E. $16x^2 + 25y^2 = 400$

F. $9x^2 + 16y^2 - 36x + 64y - 44 = 0$

Questions to ask.

1. By looking at the denominators, what does this tell you about the graph? Is it elongated along the x or y axis?
2. As the denominators become nearer the same size, what happens to the form of the ellipse? As they become very different, what happens to the shape?
3. Can you tell or explain why it is best to choose one variable over the other when solving to obtain points to plot?

GRAPHING

CONCEPT OBJECTIVE: The student will plot the points of the graph of relations.

HYPERBOLAS

PROCEDURE: Solve a given relation for either of the variables. use estimation of the irrational numbers found to find points. Find the mid-point, the asymptotes and the vertices.

PROBLEMS: Construct the graph of each of the following:

A. $\frac{x^2}{16} - \frac{y^2}{9} = 1$ B. $\frac{x^2}{36} - \frac{y^2}{9} = 1$

C. $\frac{y^2}{20} - \frac{x^2}{25} = 1$ D. $\frac{y^2}{49} - \frac{x^2}{9} = 1$

E. $16x^2 - 25y^2 = 400$ F. $9x^2 - 16y^2 - 36x - 64y - 172 = 0$

G. $xy = 16$ H. $xy = -9$

Questions to ask.

1. How do you know which way the branches of the hyperbola open?
2. Why do you only need to find points in one quadrant?

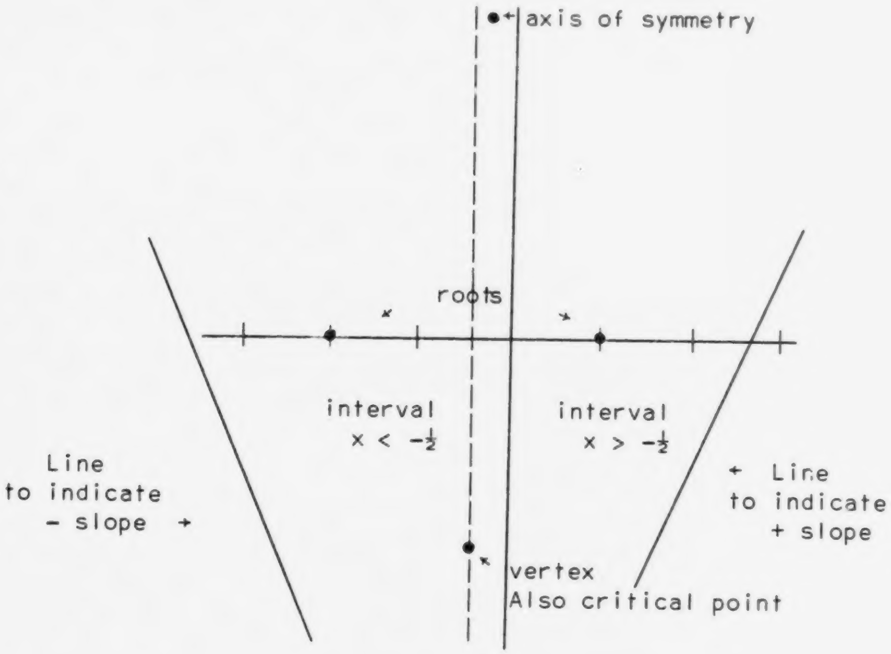
GRAPHING

CONCEPT OBJECTIVE: The student will sketch the graph of a polynomial without plotting points.

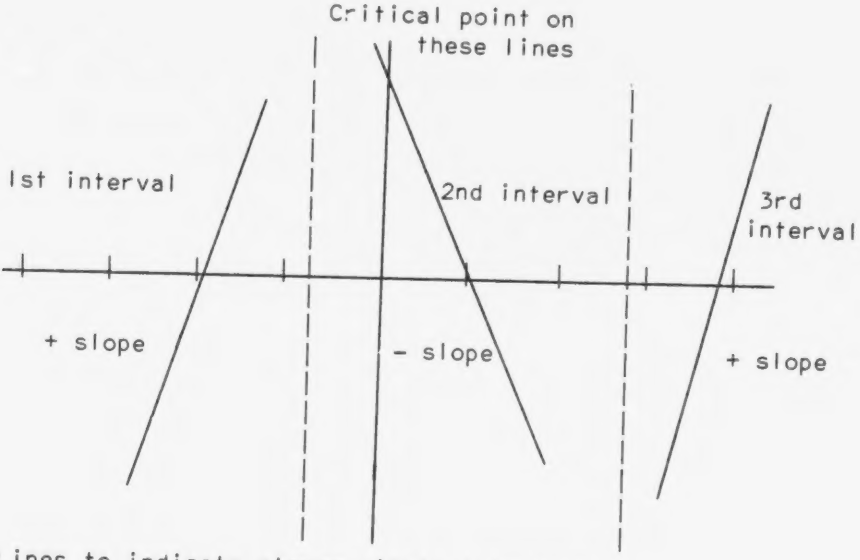
LEANING LINES

PROCEDURE: Students have by this time in their mathematical careers, graphed various lines. Functions of higher degree are easily sketched using the variety of items available to them. Using a second degree function such as $x^2 + x - 2$, students should recognize this as a parabola and find the roots immediately by factoring. The axis of symmetry can be found either by using the formula, $x = -b/2a$, or by finding the first derivative and setting it equal to zero. This will give two intervals on either side of the axis of symmetry where the slope of the line is either negative or positive.

Using the first derivative, substitute some value for x. Since the axis of symmetry in the example is $-\frac{1}{2}$, choose one value to be 1 and the other to be -1. The slope then is positive on the right side and negative on the left. See sketch below.


axis of symmetry
roots
interval
$x < -\frac{1}{2}$
interval
$x > -\frac{1}{2}$
Line
to indicate
- slope
Line
to indicate
+ slope
vertex
Also critical point

To go one step further, next use a cubic function such as $x^3 - 3x^2 - 6x + 8$. By using the rational root theorem and synthetic division, the roots can be readily found. The upper and lower bounds can also be found with this process. The first derivative is $3x^2 - 6x - 6$. The critical point, i.e., the point where the slope of the line is zero, is located at $x = 2.732$ and $x = -.732$. This gives three intervals at which one can find the slope. One interval is $x < -.732$; the second is $-.732 < x < 2,732$; and the third is $x > 2.732$. Any value within the interval will indicate the slope of the line at that point. Slopes are positive in the first and third intervals and negative in the second. See the sketch below.


Critical point on these lines
1st interval
2nd interval
3rd interval
+ slope
- slope
+ slope

Lines to indicate slopes within intervals

GRAPHING

CONCEPT OBJECTIVE: The student will either graph the equation of higher degree polynomials by plotting points or sketch the graph by using the first and second derivative.

POLYNOMIALS

PROCEDURE: With the procedure to find the intervals between critical points using the first derivative, the second derivative can be used to find inflection points, if any exist. The rational root theorem and synthetic division can be used, if necessary.

PROBLEMS: Graph the following.

1. $2t^4 + 3t^3 - 4t^2 + 6t - 5 = 0$
2. $2x^5 + x^3 + 4 = 0$

List rational roots, critical points, points of inflection, if they exist.

CONIC SECTIONS

CONCEPT OBJECTIVE: The student will find the eccentricity of the various conics, and write an equation of the conic in standard form.

ECCENTRICITY

PROCEDURE: The eccentricity of a conic is the ratio c/a, where c = focus, a = semi-major axis. The eccentricity will indicate the shape of the curve. If $e = 0$, the curve is a circle; if $e = 1$, it is a parabola; if $e > 1$, we have a hyperbola; and if $0 < e < 1$, it is an ellipse.

PROBLEMS:

1. Find the eccentricity of a conic with major axis of length 10 with foci at (0, 4) and (0, -4). Write the equation in standard form.

2. Find the eccentricity and write an equation for the conic if the minor axis has length 6, foci at (2, 0) and (-2, 0).

CONIC SECTIONS

CONCEPT OBJECTIVE: The student will graph an ellipse.

GRAPHING AN ELLIPSE

PROCEDURE: Identify the locus of $16x^2 + 9y^2 - 64x - 54y + 1 = 0$, and determine its principal characteristics.

Complete the square in x and y. The equation is that of an ellipse $\frac{(x-2)^2}{9} + \frac{(y-3)^2}{16} = 1$. The center of the ellipse is at (2, 3), length of the major axis is 8, the length of the minor axis is 6. The eccentricity is $\sqrt{7}/4$ foci located at $(2, 3 \pm \sqrt{7})$. The end of the minor axis (5, 3) and (-1,3); end of the major axis (2, 7) and (2, -1). To sketch the graph of this relation, plot the center and endpoints. Construct a "box" into which the ellipse will be by sketching vertical and horizontal lines through the endpoints of the major and minor axes. Because of symmetry, plotting a few points in the first quadrant, the graph can be completed quickly.

CONIC SECTIONS

CONCEPT OBJECTIVE: The student will graph a hyperbola.

GRAPHING A HYPERBOLA

PROCEDURE: Identify the locus of $16x^2 - 9y^2 - 64x + 54y - 161 = 0$, determine its principal characteristics and sketch the graph.

Complete the square to obtain the more useful equation $\frac{(y-3)^2}{16} - \frac{(x-2)^2}{9} = 1$. The center of this hyperbola is at (2, 3), with $a = 4$ and $b = 3$. The major axis is parallel to the y-axis. The length of the major axis is 8 and the length of the minor axis is 6. The vertices are located at (2, 7) and (2, −1). We know the point of intersection of the graph and the minor axis is imaginary. However, if there was a point, where would it be located? By taking b and adding to and subtracting from the y-coordinate, these points would be (−1, 3) and (5, 3). As with the ellipse, construct a "box". By connecting the opposite corners of this box we have constructed the asymptotes. Use the center and two corners, the equation of the asymptotes can be derived as $y - 3 = \pm\frac{4}{3}(x - 2)$. We know that the graph of the hyperbola cannot be inside the box. By plotting a few points and using symmetry, the graph can be quickly sketched. Foci located at (2, 8) and (2, −2). $e = \frac{5}{4}$

CONIC SECTIONS

CONCEPT OBJECTIVE: The student will practice solving equations satisfying given geometric conditions.

EQUATIONS

PROCEDURE: Following the discussion and work with the conic section along with graphing, define locus as follows:

A plane curve is the locus of all those points in a place, and only those points, which satisfy a given geometric condition.

To find the equation of the locus:

A. Let $P\ (x, y)$ be any point on the locus.

B. Express the given geometric conditions by means of an equation in x and y.

C. Simplify the equation obtained in B.

Equations may be identified as to shape and/or graphed.

PROBLEMS:

1. Find the equation of the locus of a point which moves so that the slope of the line joining it to $A(-3, -2)$ is twice that of the line joining it to the point $B(-5, -8)$.
2. Find the equation of the locus of a point which moves so that the sum of the squares of its distances from the points $A(-2, -2)$ and $B(1, 1)$ is always equal to 3.
3. Find the equation of the locus of a point which moves so that the sum of its distances from the points $A(0, 5)$ and $B(0, -5)$ is 12.
4. Find the equation of the locus of the center of a circle which passes through the point $A(-2, 5)$ and is tangent to the line $y = 2$.

COMPLEX NUMBERS

CONCEPT OBJECTIVE: The student will find the square root of complex numbers.

SQUARE ROOT

PROCEDURE: Given some complex number such as $2 - 2i\sqrt{3}$. Find the factors that will give this number when either factor is squared.

By definition $|(a + bi)|^2 = a^2 + b^2$

By definition the modulus $|a + bi| = \sqrt{a^2 + b^2}$

The real part of $2 - 2i\sqrt{3}$ is 2.

The imaginary part is $2\sqrt{3}$.

$(a + bi)^2 = a^2 - b^2 + 2abi = 2 - 2i\sqrt{3}$

$$a^2 - b^2 = 2$$

$$2ab = -2\sqrt{3}$$

$$\begin{aligned} |a + bi|^2 = 2 - 2i\sqrt{3} &= \sqrt{2^2 + (-2\sqrt{3})^2} \\ &= \sqrt{4 + 12} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

$$\begin{array}{r} a^2 + b^2 = 4 \\ \underline{a^2 - b^2 = 2} \\ 2a^2 = 6 \\ a^2 = 3 \\ a = \pm\sqrt{3} \\ b = \pm 1 \end{array}$$

Therefore, the roots of $2 - 2i\sqrt{3}$ are $\sqrt{3} - i$ and $-\sqrt{3} + i$

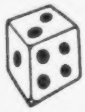
PROBLEMS: Find the solutions.

1. $x^2 = 2i$

2. $y^2 = 7 - 24i$

3. $x^2 = -21 + 20i$

CONCEPT OBJECTIVE: The student will practice finding vector length.



COMPLEXITY

PROCEDURE: Each group of 2-6 students will need a deck of Complexity cards. After cards are shuffled, 5 cards are dealt to each player. The rest of the cards are placed face down in a stack before the dealer to be used in trading. A player looks at his/her hand and attempts to use 4 of the cards dealt to form a complex number of the form $a \pm bi$, where $a, b \neq 0$. Beginning with the player on the left of the dealer, each player may choose to trade any or all of his/her cards one time. After trading, each lays down the best possible complex number, using 4 of the 5 cards from his/her hand.

Vector length, $\sqrt{a^2 + b^2}$, is determined for each complex number. If a player cannot make a number in the form $a \pm bi$, he/she has no score for that round. The winner of the hand is the player with the largest vector length. The player winning the most hands at the end of playing time is the winner of the game.

Complexity cards are composed of digit cards, operation cards, and i cards. A deck of 40 cards has 18 digit cards (two each of the digits 1-9), 10 operation cards, (five each of plus and minus), and 12 cards with i.

POLYNOMIAL EQUATIONS

CONCEPT OBJECTIVE: The student will write the equation of a polynomial using the relationship between the roots and coefficients.

FINDING EQUATIONS

PROCEDURE: Have the students review the use of the sums and products of the roots of an equation. Have them recall the sum of the roots of a quadratic equation is the inverse of the coefficient of the middle term and the product of the roots gives the constant term. This works fine as long as the leading coefficient is one. It leads to the following:

If $a_0x^n + a_1x^{n-1} + a_2x^{n-2} = ... + a_n = 0$ $(a_0 \neq 0)$

has roots $r_1, r_2, r_3, ... r_n$ in C (complex numbers) the sum of the roots taken k at a time is $(-1)^k$ times the product of the coefficient of x^{n-k} or even number terms are opposite in sign of the sum. If a_0 is not one, the polynomial must be divided by a_0 in order to make the leading coefficient 1. Letting $S_1, S_2, . . . S_n$ be the sum of each coefficient starting with the second term we have

$$S_1 = r_1 + r_2 + ... r_n$$

$$S_2 = r_1r_2 + r_1r_3 + r_2r_3 + ... r_{n-1}r_n$$

$$S_3 = r_1r_2r_3 + r_1r_2r_{n-1} + r_1r_2r_n + r_2r_3r_{n-1} + ... + r_{n-2}r_{n-1}r_n$$

$$S_n = r_1r_2r_3 . . . r_n$$

To make sure you have all the combinations for each coefficient use the factorial form (n_k).

Example: Find the equation with roots as follows:

$r_1 = 2, r_2 = -3, r_3 = 1$

$S_1 = 2 + (-3) + 1 = 0$

$S_2 = 2(-3) + 2(1) + (-3)1 = -7$

$S_3 = 2(-3)1 = -6$

The equation is $x^3 + 0x^2 - 7x + 6 = 0$

PROBLEMS: Find the equations given the following information:

1. Roots are $1, i, -i$

2. Roots are $3, -2, 1, 0$

3. Roots are $2 + i, 2 - i, 4$

4. $x + 2$ and $x - 3$ are double factors

5. The sum of three roots of $2x^4 - 1x^3 - 11x + 4x + 12 = 0$ is $\frac{3}{2}$. Two of these roots are inverses of each other. Find the roots.

APPLICATIONS OF THE DERIVATIVE

CONCEPT OBJECTIVE: The student will use the first derivative to solve problems.

BY THE NUMBERS

PROCEDURE: Review the use of the derivative. Remind students the critical point on a curve is the point at which the slope of the tangent is zero and the derivative can be used to find the slope of a line.

PROBLEMS:

1. The sum of two positive numbers is 18. Find the numbers if the sum of their squares is a minimum.
2. The sum of two positive numbers is 18. Find the numbers if the product of one and the square of the other is a maximum.

SOLUTIONS:

Let x = first number; $18 - x$ the second number.

1. $x^2 + (18 - x)^2 = x^2 + 324 - 36x + x^2 = 2x^2 - 36x + 324$

$$f'(x) = 4x - 36$$

$$4x - 36 = 0$$

$$4x = 36$$

$$x = 9$$

2. $x(18 - x)^2 = x(324 - 36x + x^2) = 324x - 36x^2 + x^3$

$$f'(x) = 324 - 72x + 3x^2$$

To simplify, divide by $3 = 108 - 24x + x^2$

$$x^2 - 24x + 108 = 0$$

$$(x - 6)(x - 18) = 0$$

$$x = 6,\ 18$$

APPLICATIONS OF THE DERIVATIVE

CONCEPT OBJECTIVE: The student will use the derivative to solve problems involving maxima and minima.

ROUND 2

PROCEDURE: The student will be acquainted with the first derivative.

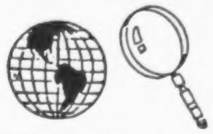
PROBLEM:

A shell is fired vertically into the air from a cliff 176 feet above the surface of water. If the initial velocity of the shell is 1,280 feet per second, and the formula for finding the altitude is $a = h + vt - 16t^2$, what is the highest point the shell will attain, and how long will it take for the shell to reach this point?

a. How much time will have elapsed when the shell hits the water?

b. How much time will have elapsed when the shell reaches the 20,000-ft. level?

CONCEPT OBJECTIVE: The student will practice using the first derivative to solve problems.



ENCLOSURES

PROCEDURE: Review the first derivative and its uses for finding maxima and minima.

PROBLEM:

A rancher wishes to enclose a plot of land on three sides. He has at his disposal 1,600 feet of fencing. What is the maximum area that can be enclosed?

APPLICATIONS OF THE DERIVATIVE

CONCEPT OBJECTIVE: The student will use the derivative to solve problems involving maxima and minima.

FIRE ONE

PROCEDURE: Review students on the use of the derivative and the various formulas needed for maxima and minima.

The formula for finding the altitude a missile attains is given as $a = vt - 16t^2$, v = velocity and t = time.

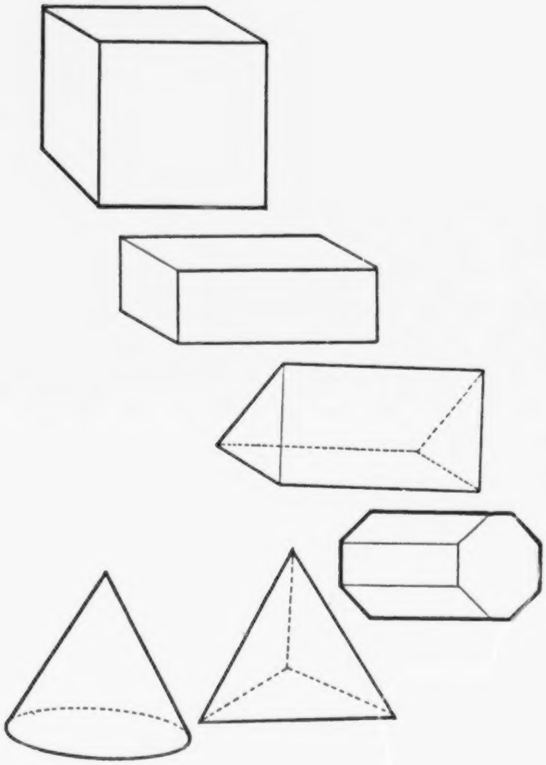
PROBLEMS:

1. If a missile is fired vertically with an initial velocity of 1,440 feet per second, how much time will it take to reach its highest point?
    a. What is the maximum altitude this missile reaches?
    b. At what time will the missile be at 20,000 feet? (Note: round off to the nearest full second.)
    c. How many seconds will it take to return to earth?
2. How much change in time would there be if the missile in the problem above was fired from a platform 200 feet above the ground?

RESOURCES

PEOPLE, BOOKS, AND MORE

ASSISTANCE FOR THE EDUCATOR



RESOURCES

MATHEMATICS CONTESTS

The OKLAHOMA MATHEMATICS LEAGUE offers students in Oklahoma high schools the opportunity to participate in interscholastic mathematics activities at the school, county, and state levels. A school's math team consists of five or more students who take six short answer examinations during the school year. Awards are given at all three levels of competition.

The League also offers contests at the junior high/middle and elementary levels.

CONTACT: Oklahoma Mathematics League
P.O. Box 3136
Bartlesville, OK 74005

The ANNUAL HIGH SCHOOL MATHEMATICS EXAMINATION is a test given to students throughout the world. Its purpose is to create and sustain interest in mathematics by developing sound scholarship in the subject, providing challenging problems which teach. The examination will broaden and deepen the student's experience with mathematical concepts.

The test consists of 30 questions to be completed in 90 minutes. The student having the best score from a participating school will receive an award. All students achieving a set standard will be recognized. The United States 8-member team is selected from finalists on this examination to represent the nation in the International Olympiad Competition.

CONTACT: Dr. Tom Cairns
Mathematics Department
University of Tulsa
600 South College
Tulsa, OK 74104

The ATLANTIC - PACIFIC HIGH SCHOOL MATHEMATICS LEAGUE offers students in high schools throughout the United States and Canada an opportunity to compete in a series of math contests. There are six contests scheduled from October to April. Each contest, to be given at the school under the supervision of the sponsor, consists of six problems to be completed within a 30-minute time limit. A team's score will be the scores of its ten highest-scoring students for that contest. The questions will be drawn from all areas of high school mathematics through pre-calculus.

CONTACT: Atlantic - Pacific High School
Mathematics League
P.O. Box 11242
Elkins Park, PA 19117

MATHCOUNTS is a national math contest for seventh and eighth grade students. The program is designed to encourage achievement in mathematics, increase awareness of the importance of mathematics, and assist in the improvement of mathematics curriculum and instruction. A series of contests are held beginning at the school level and continuing with regional and state competition. A four-member team, comprised of the top individual scorers in the state contest, represent the state in the national competition.

CONTACT: Oklahoma Society of Professional Engineers
Oklahoma Engineering Center
201 Northeast 27 Street
Oklahoma City, OK 73105

STUDENT ORGANIZATIONS

MU ALPHA THETA

MU ALPHA THETA is a mathematics club for senior high school and junior college students cosponsored by the Mathematics Association of America and the National Council of Teachers of Mathematics. The club's purpose is to stimulate interest in mathematics among students by providing public recognition of superior mathematical scholarship and promoting mathematical activities. The organization's journal, *The Mathematical Log*, contains up-to-date understandable articles on mathematics.

CONTACT: Dr. Harold Huneke
Mathematics Department
University of Oklahoma
Norman, OK 73019

CHI ALPHA MU

CHI ALPHA MU is a mathematics club for junior high/middle school students. The purpose of Chi Alpha Mu is to promote interest in, and enjoyment and understanding of mathematics among students. To qualify for membership, a school must have a mathematics department of at least one certified mathematics teacher during the preceding school year.

CONTACT: Mary E. Lester
Chi Alpha Mu Secretary
5000 S. Oakland Ave.
Dallas, TX 75215

TEACHER ORGANIZATIONS

The Mathematical Association of America
1529 Eighteen Street NW
Washington, D.C. 20036

National Council of Teachers of Mathematics
1906 Association Drive
Reston, VA 22091

Oklahoma Council of Teachers of Mathematics
c/o Mathematics Specialist
State Department of Education
2500 N. Lincoln
Oklahoma City, OK 73105-4599

School Science and Mathematics Association
126 Life Science Building
Bowling Green State University
Bowling Green, OH 43403

PERIODICALS

The Arithmetic Teacher
National Council of Teachers of Mathematics
1906 Association Drive
Reston, VA 22091

The Mathematics Teacher
National Council of Teachers of Mathematics
1906 Association Drive
Reston, VA 22091

Journal of Recreational Mathematics
Baywood Publishing Co., Inc.
43 Central Drive
Farmingdale, NY 11735

Scholastic Math Magazine
2280 Arbor Blvd.
Dayton, OH 45430

School Science and Mathematics
School Science and Mathematics Assn. Inc.
Box 246
Bloomington, IN 47401

Science 84
American Association for the Advancement of Science
1515 Massachusetts Ave. NW
Washington, D.C. 20005

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Middletown, CT 06457

GAMES

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Challenge Yahtzee. Milton Bradley Co., 443 Shaker Rd., Longmeadow, MA 01028

Chinese Checkers. Ohio Art, P.O. Box 111, Bryan, OH 43506

Impuzzables. Creative Publications, Inc., P.O. Box 13028, Palo Alto, CA 94303

Mastermind. Invicta Plastics (USA) Ltd., 200 5th Ave., New York, NY 10010

Tengo. Dual Puzzles Co., Creative Publications, Inc., P.O. Box 13028, Palo Alto, CA 94303

GOVERNMENT AND BUSINESS SUPPLEMENTAL MATERIALS

How to Choose and Use Retail Credit
Consumer Information Services
Dept. 703
Sears, Roebuck & Co.
Sears Tower
Chicago, IL 60684

Mathematics in the Petroleum Industry
American Petroleum Institute
1271 Avenue of the Americas
New York, NY 10020

Understanding Taxes
Internal Revenue Service
P.O. Box 66
Oklahoma City, OK 73102

Fundamental Facts About United States Money
Research Department
Federal Reserve Bank of Atlanta
Atlanta, GA 30303

Sets Probability and Statistics: The Mathematics of Life Insurance
Educational Division
Institute of Life Insurance
277 Park Avenue
New York, NY 10017

National Aeronautics and Space Administration
Lyndon B. Johnson Space Center
Houston, TX 77058

MISCELLANEOUS

CHIME Newsletter
Oklahoma State University
108 Gunderson
STillwater, OK 74078

"Mathematics at Work in Society: a mathematics and career awareness package"

The Mathematical Association of America
1529 Eighteenth St. NW
Washington, D.C. 20036

OSU MAWIS Program
Office of Student Services
201 Whitehurst
Stillwater, OK 74078

Oklahoma Society of Professional Engineers
Oklahoma Engineering Center
201 NE 27 Street
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State Department of Transportation
200 NE 21 Street
Oklahoma City, OK 73105

"Visiting Mathematics Lecturer Program"
Dr. Robert McMillan
Department of Mathematics
Oklahoma Christian College
Oklahoma City, OK 73111

ENRICHMENT

In an attempt to meet the needs of each individual student, a teacher may rely on a variety of resources. The following are ideas for mathematics enrichment. Some are beginning suggestions which can be extended and adapted by the teacher. While some of the activities are appropriate for individual exploration, others could be used with groups of students.

AFTERS

An "AFTERS" box can provide students who finish early an opportunity to broaden their mathematical abilities. After gathering several activity ideas and duplicating copies, prepare an envelope by placing a copy of the activity on the front and covering it with clear adhesive-backed plastic. In the envelope, place the remaining copies of the activity. Envelopes may be placed in a decorated box where students will have access to it. Suggestions for the activity box include crossword puzzles, word searches, and logic puzzles.

Guided manipulation of numbers and the use of arithmetic operations can provide students with challenges and the discovery of interesting relationships. Both of the following problems offer students the opportunity for individual exploration.

PERFECT NUMBERS

A number is perfect if it is the sum of its proper divisors. For example, 6 is perfect because $1 + 2 + 3 = 6$. Students can identify other perfect numbers and investigate the following:

Is there an odd perfect number?

Determine the sum of the reciprocals of all the divisors of a perfect number.

Is the sum of the reciprocals of all the divisors the same for all perfect numbers?

1985

Students can select any year, such as 1985, and try to form the numbers 1 to 100 using the following restrictions:

1. Use only the digits in the year in the order in which they appear in the year.
2. Any of the following operations may be used as often as needed:

   Addition
   Subtraction
   Multiplication
   Division
   Exponents
   Square root
   Factorial

3. Digits may be juxtaposed to form 2 or 3-digit numbers.
4. Parentheses may be used as needed.

As an example, selecting the year, 1985:

$1^9 + (8 \times 5) = 41$

$1 - \sqrt{9} + 85 = 83$

A number of word mazes exist for a variety of mathematical topics. A maze can provide an interesting way to review vocabulary terms. Students can also create their own mazes to share with others.

This word maze contains over 60 terms connected with the study of geometry. The words may be read horizontally, vertically, or diagonally, and often in reverse order.

GEOMETRY MAZE

G	E	O	M	E	T	R	Y	A	B	C	S	U	R	O	G	A	H	T	Y	P	D	E	F	W
R	E	Q	P	M	O	N	M	L	C	K	Q	H	E	I	G	H	T	J	I	H	G	T	N	B
S	Q	T	U	E	S	U	T	B	O	V	U	S	S	A	P	M	O	C	W	G	X	R	E	Q
E	U	C	N	A	O	W	O	N	N	Z	A	T	N	E	G	N	A	T	Y	N	E	A	E	W
A	I	S	O	S	C	E	L	E	S	B	R	S	G	P	L	A	N	E	A	I	S	P	W	U
M	L	E	B	U	C	I	T	T	T	S	E	E	T	I	C	I	N	I	E	D	U	E	T	O
E	A	F	O	R	M	H	E	T	R	O	T	G	R	A	O	I	N	E	T	N	N	Z	E	L
C	T	D	N	E	G	V	A	N	U	U	O	A	E	P	M	L	R	E	T	O	E	O	B	Y
O	E	O	Y	I	N	E	J	U	C	S	T	N	T	O	P	G	A	C	F	P	T	I	I	D
N	R	T	R	S	N	R	E	A	T	H	T	T	P	S	E	U	Y	L	L	S	O	D	L	Y
C	A	E	A	E	S	T	R	A	I	G	H	T	U	D	M	E	M	P	P	E	P	T	R	G
E	L	T	T	E	E	I	E	P	O	S	E	B	R	A	E	U	Q	S	E	R	Y	Y	A	O
N	E	R	N	R	G	C	D	R	N	I	M	O	D	I	N	F	I	P	N	R	H	L	L	L
T	V	A	E	O	M	A	T	E	I	O	N	P	R	O	T	R	A	C	T	O	R	A	U	O
R	E	H	M	M	E	L	A	C	H	O	R	D	K	E	A	A	L	N	A	C	Y	M	C	P
T	R	E	E	H	N	L	L	R	I	A	R	M	E	M	R	O	T	C	G	E	R	O	I	O
C	T	D	L	N	T	N	E	U	R	G	N	O	C	A	Y	N	E	E	O	M	Y	S	D	T
H	E	R	P	G	E	R	E	D	E	L	G	N	A	T	C	E	R	S	N	F	L	E	N	D
O	X	O	P	T	N	L	I	M	E	W	E	L	P	L	G	O	N	M	M	A	N	I	E	P
S	A	N	U	D	Y	A	A	A	U	C	L	L	S	A	R	E	A	E	R	T	O	O	P	O
H	T	H	S	A	M	T	I	C	N	L	A	R	O	I	R	E	T	X	E	P	S	A	R	L
N	A	I	D	E	M	S	S	I	S	G	O	G	E	R	P	A	D	X	D	E	N	E	E	Y
I	O	N	T	U	S	H	E	D	D	W	L	V	O	H	E	N	L	L	Y	O	T	M	P	G
L	E	E	T	A	E	H	N	O	G	A	X	E	H	R	N	U	M	L	E	N	G	T	H	O
S	R	E	T	R	R	E	T	E	M	I	R	E	P	W	E	G	I	F	E	T	N	A	C	N
O	S	A	E	Y	K	A	C	I	R	C	U	M	F	E	R	E	N	C	E	L	R	E	N	X

Often a practical application of mathematics can motivate students to practice skills. One application may involve scientists who use formulas to assist in determining the identity of a crime victim.

BONES, BONES

The exact physical dimensions of a victim of a crime are very useful in identifying the victim. When a skeleton is found, the length of certain bones is used to calculate the height of a living person. The bones used are the femur (F), the tibia (T), the humerus (H), and the radius (R).

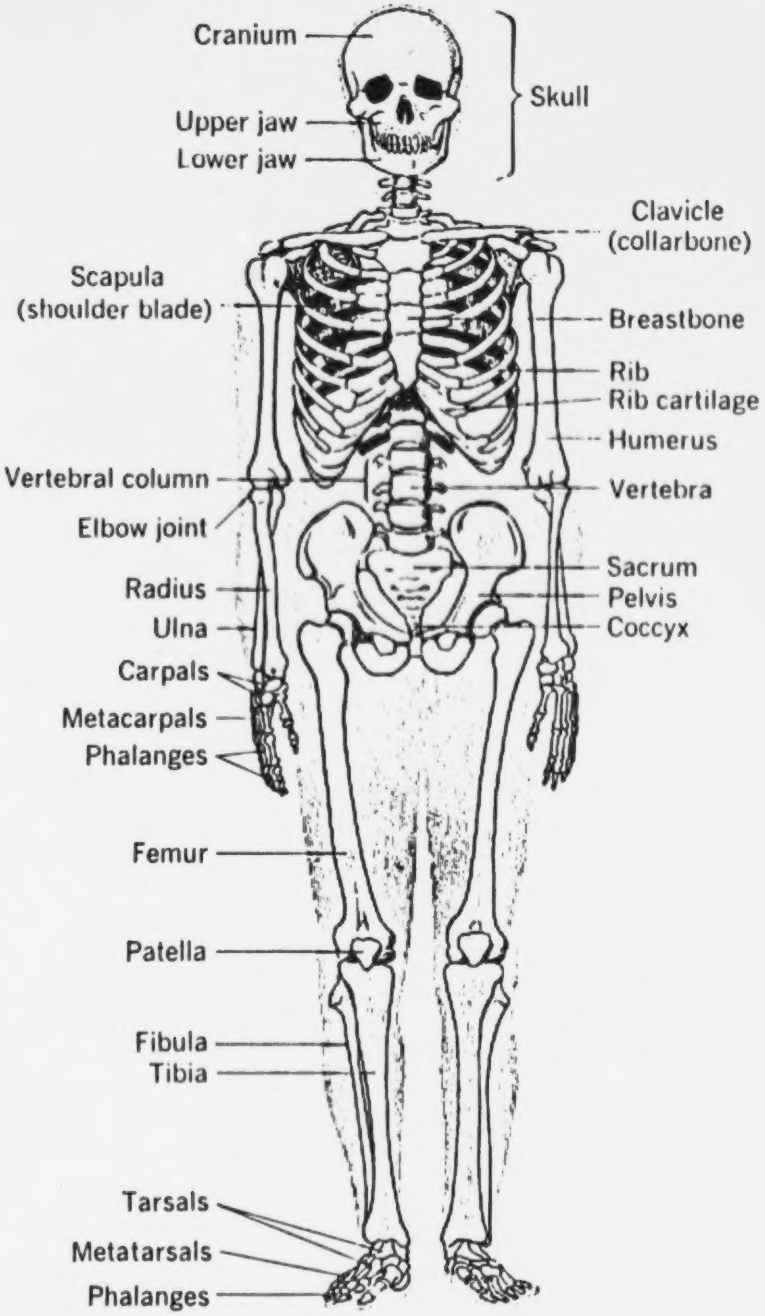
A formula has been devised by forensic scientists to assist in determining the height of a person.

MALE	FEMALE
$h = 69.089 + 2.238F$	$h = 61.412 + 2.317F$
$h = 81.688 + 2.392T$	$h = 72.572 + 2.533T$
$h = 73.570 + 2.970H$	$h = 64.977 + 3.144H$
$h = 80.405 + 3.650R$	$h = 73.502 + 3.876R$

It must be noted that at approximate age 30, the height of a person begins to decrease at the rate of 0.06 mm per year. This must be taken into consideration when the age of a victim is known.

PROBLEMS:

1. Have students use the formula to calculate the lengths of their femus, tibia, humerus, and radius.
2. Have students contact the local law enforcement agencies to determine how mathematics is used in crime detection.
3. What is the height of a 27-year-old male whose tibia measures 34.5 cm?


Cranium
Upper jaw
Lower jaw
Skull
Clavicle (collarbone)
Scapula (shoulder blade)
Breastbone
Rib
Rib cartilage
Humerus
Vertebral column
Vertebra
Elbow joint
Sacrum
Radius
Pelvis
Ulna
Coccyx
Carpals
Metacarpals
Phalanges
Femur
Patella
Fibula
Tibia
Tarsals
Metatarsals
Phalanges

Reprinted with permission: FUNDAMENTAL CONCEPTS OF BIOLOGY, Sebastian Haskel and David Sygoda, AMSCO School Publications, Inc., 1972.

Puzzles, such as the following, help students develop skills in organizing information, reasoning and logical deduction. A variety of similar puzzles exist which provide students with interesting challenges in problem solving.

FIVE TOOL BOXES

Students should use the following information to answer the questions at the end.

There are five tool boxes, each a different color, and belonging to individual craftsmen of five different crafts. Each craftsman has a different tool, drink, and birthday month.

1. The electrician has the red tool box.
2. The carpenter has the wrench.
3. Coffee is on the green tool box.
4. The plumber drinks tea.
5. The green tool box is immediately to the right of the black tool box.
6. The owner of the yellow tool box was born in July.
7. The screwdriver belongs to the craftsman born in October.
8. Milk is on the middle tool box.
9. The mechanic has the first tool box on the left.
10. The craftsman born in March has the tool box that is next to the tool box with the hammer.
11. The craftsman born in July has the tool box next to the tool box with the pliers.
12. The craftsman born in November drinks orange juice.
13. The painter has his birthday in April.
14. The mechanic's tool box is next to the blue tool box.
15. The owner of the black tool box drinks orange juice.
16. The tool box with the hammer is four tool boxes from the tool box owned by the craftsman born in April.

WHICH CRAFTSMAN DRINKS WATER? WHICH CRAFTSMAN HAS THE TAPE MEASURE?

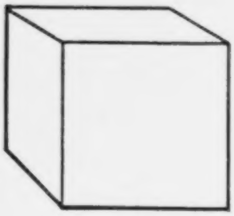
Student projects can provide an opportunity for individual exploration and creativity. Projects can be required assignments or outside enrichment activities.

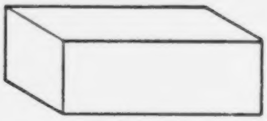
STRING ART

String art designs can be created while skills in measurement and construction are practiced. Students can develop variations on particular designs.

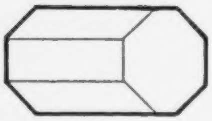
POSTERS

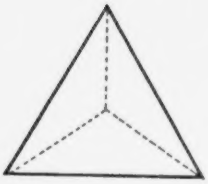
A poster project can promote exploration in a variety of topics. A number of categories, such as mathematics used in a career, contributions of a famous mathematician, the role of mathematics in games, and mathematics used in computer programming, can be presented. Students would make a selection and construct a poster to illustrate their topic.

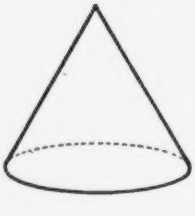




APPENDIX

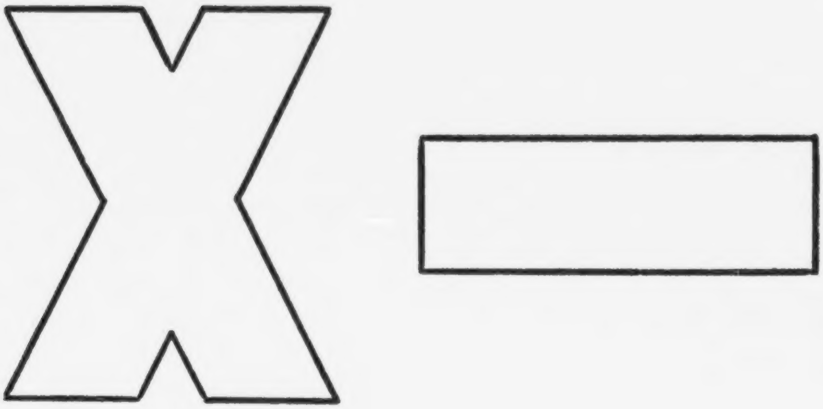






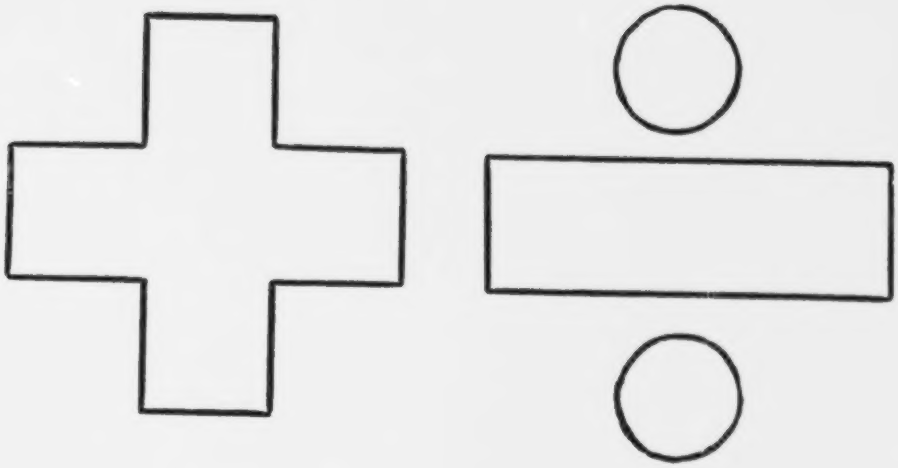
9	10	11	12	13	14	15	16

COVER 16



1	2	3	4	5	6	7	8

1	2	3	4	5	6	7	8

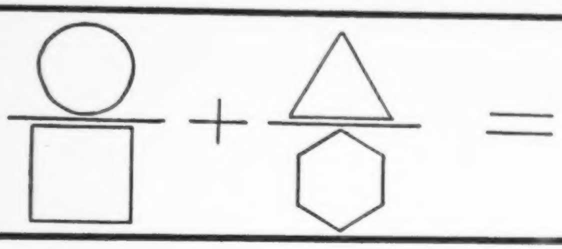


COVER 16

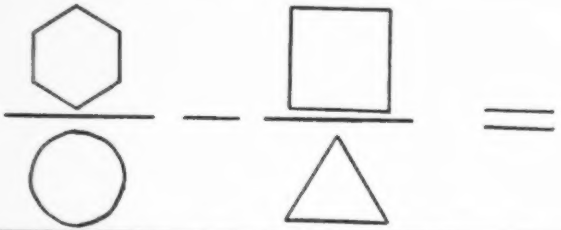
9	10	11	12	13	14	15	16

FRACTION ROUNDS

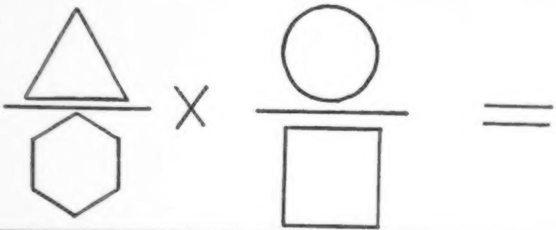
Round 1



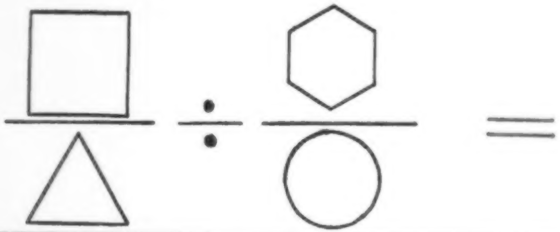
Round 2



Round 3



Round 4



TOTAL

STUDENT INFORMATION SURVEY

Name________________________________

1. Age________

2. Color of hair________

3. Color of eyes________

4. Month of birth________

5. Year of birth________

6. Favorite color:

blue green
red purple
orange yellow

7. Shoe size________

8. How many pets do you have?

0 1 2 3 or more

9. Which do you prefer?

Coca-Cola Pepsi

10. Elementary school last attended:

Will Rogers Skyline
Highland Park Westwood
Other

11. Time spent watching TV last night:

0 1 or more hours
<1 hour 2 or more hours
3 or more hours

12. Favorite elective subject________

13. Favorite sport of these listed:

Football Wrestling
Basketball Baseball

14. Total brothers and sisters:

0 1 2 3 4 or more

SCORE CARD

Hole	Name:	Name:	Course Rules
	Score:	Score:	
1			The sum is positive.
2			The sum is -2.
3			The product is greater than -12.
4			The sum is divisible by 2.
5			The sum is 3.
6			The product is divisible by 3.
7			The sum is negative.
8			The product is less than -18.
9			The sum is zero.
	Total:	Total:	

THE GREATEST VALUE

GAME I

$$(\triangle X + \bigcirc)(\square X + ⬡)$$

GAME II

$$(\square X - \triangle)(\bigcirc X + ⬡)$$

THE GREATEST VALUE

GAME III

□ (⬡ X + △) (X + ○)

GAME IV

⬡ (X + △) (X + ○) (X + □)

FACTORING DOMINOES

A set of 32 dominoes has been created pairing each trinomial randomly with four different factored expressions.

$X^2+8X+12$	$(X-3)(X+2)$	$X^2+8X+12$	$(X+4)(X-2)$	$X^2+8X+12$	$(X-10)(X+4)$
$X^2+8X+12$	$(X-9)(X+3)$	X^2-X-6	$(X+6)(X+2)$	X^2-X-6	$(X+8)(X+3)$
X^2-X-6	$(X+8)(X-2)$	X^2-X-6	$(X-10)(X+4$	$X^2-9X+18$	$(X+6)(X+2)$
$X^2-9X+18$	$(X+4)(X-2)$	$X^2-9X+18$	$(X+8)(X+3)$	$X^2-9X+18$	$(X+8)(X-2)$
X^2+2X-8	$(X-6)(X-3)$	X^2+2X-8	$(X-3)(X+2)$	X^2+2X-8	$(X-9)(X+3)$
$X^2-6X-27$	$(X-6)(X-3)$	$X^2-6X-27$	$(X-3)(X+2)$	$X^2-6X-27$	$(X+4)(X-2)$
X^2+2X-8	$(X-10)(X+4)$	$X^2-6X-27$	$(X+8)(X-2)$	$X^2+11X+24$	$(X+6)(X+2)$
$X^2+11X+24$	$(X-6)(X-3)$	$X^2+11X+24$	$(X-9)(X+3)$	$X^2+11X+24$	$(X-3)(X+2)$
$X^2-6X-40$	$(X+8)(X+3)$	$X^2-6X-40$	$(X+6)(X+2)$	$X^2+6X-16$	$(X-10)(X+4)$
$X^2+6X-16$	$(X-6)(X-3)$	$X^2-6X-40$	$(X+8)(X-2)$	$X^2-6X-40$	$(X-9)(X+3)$
$X^2+6X-16$	$(X+4)(X-2)$	$X^2+6X-16$	$(X+8)(X+3)$		

100 PROOF GIN

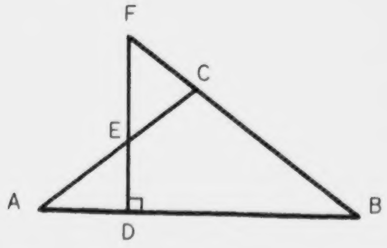
A sample list of PROOF cards.

PROOF CARDS

GIVEN: $\overline{FD} \perp \overline{AB}$

$\measuredangle A \cong \measuredangle B$

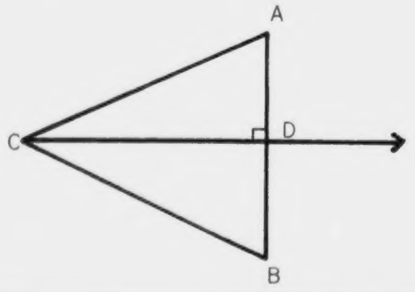
PROVE: $\measuredangle F \cong \measuredangle CEF$



GIVEN: $\measuredangle A \cong \measuredangle B$

$\overrightarrow{CD} \perp \overline{AB}$

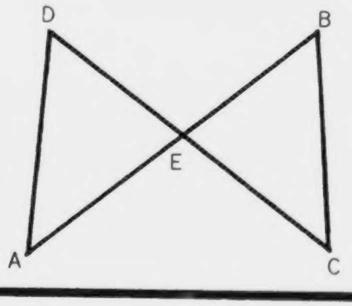
PROVE: $\overrightarrow{CD}$ bisects $\measuredangle ACB$



GIVEN: $\overline{AE} \cong \overline{CE}$

$\overline{DE} \cong \overline{BE}$

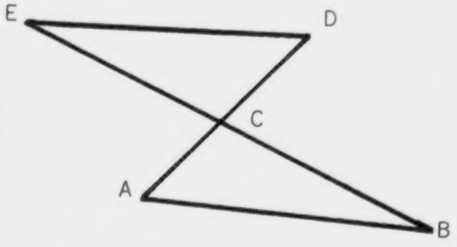
PROVE: $\triangle ADE \cong \triangle CBE$



GIVEN: C is the midpoint of

$\overline{AD}$ and $\overline{EB}$, $\measuredangle B \cong \measuredangle E$

PROVE: $\triangle ABC \cong \triangle DEC$



STATEMENTS CARDS

For this set of PROOFS cards, a corresponding set of STATEMENTS cards is composed of:

$\overline{FD} \perp \overline{AB}$
$\measuredangle A \cong \measuredangle B$

$\measuredangle AED \cong \measuredangle F$

$\measuredangle FDA \cong \measuredangle FDB$

$\measuredangle AED \cong \measuredangle CEF$

$\measuredangle F \cong \measuredangle CEF$

$\measuredangle A \cong \measuredangle B$
$\overrightarrow{CD} \perp \overline{AB}$

$\measuredangle CDA \cong \measuredangle CDB$

$\measuredangle ACD \cong \measuredangle BCD$

$\overrightarrow{CD}$ bisects $\measuredangle ACB$

$\overline{AE} \cong \overline{CE}$
$\overline{DE} \cong \overline{BE}$

$\measuredangle DEA \cong \measuredangle BEC$

$\triangle ADE \cong \triangle CBE$

C is the midpoint of $\overline{AD}$ and $\overline{EB}$
$\measuredangle B \cong \measuredangle E$

$\measuredangle ECD \cong \measuredangle ACB$

$\triangle ABC \cong \triangle DEC$

$\overline{EC} \cong \overline{BC}$
$\overline{AC} \cong \overline{CD}$

REASONS CARDS

For this set of PROOFS cards, a corresponding set of REASONS cards is composed of:

Five copies of:

"Given"

Four copies each of:

"Vertical angles are congruent"
"Two perpendicular lines form 4 congruent right angles"

Three copies of:

"If 2 angles of one triangle are congruent to 2 angles of a second triangle, then the third angles are congruent"

Two copies each of:

"Substitution"
"SAS"

and

One copy each of:

"Definition of angle bisector"
"Definition of midpoint"

GEOMETRY BINGO

For the sample Geometry Bingo game board, a corresponding set of question cards is composed of:

Find the square showing the division property of equality.

Find the examples of corresponding angles.

Find the square containing the subtraction property of equality.

Which square shows adjacent angles?

Find the square containing supplementary angles.

Find the square showing the addition property of equality.

Find the square concerning an equilateral triangle.

Find the substitution property.

Find the illustration of alternate exterior angles.

Find the Angle Addition Postulate.

What are the three undefined terms of geometry?

Find the square illustrating the reflexive property.

What is the symbol for "congruent to"?

Find the square containing complementary angles.

Find the illustration showing vertical angles.

Find the square that shows the definition of "between".

What is the symbol for an angle?

Find the illustration containing alternate interior angles.

Find the symbol for "is perpendicular to".

Find the square that describes a scalene triangle.

Find the definition or illustration of an obtuse angle.

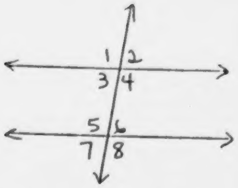
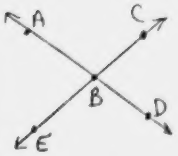
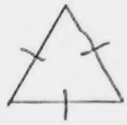
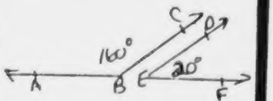
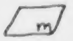
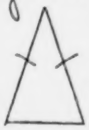
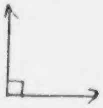
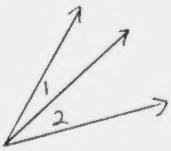
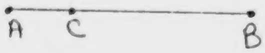
Find the definition or illustration of an acute angle.

Find the transitive property.

Find the definition of a right angle.

Find the square showing the multiplication property of equality.

GEOMETRY BINGO

 ∡3 and ∡6	 ∡ABE and ∡CBD	 A triangle with 3 congruent sides	An angle with a degree measure between 0° – 90°.
BONUS	 m∡ABC + m∡DEF = 180°	Point · Line ⟷ Plane 	≅
A triangle with at least 2 congruent sides 	For any angle A, ∡A ≅ ∡A. For any segment $\overline{AB}$, $\overline{AB} \cong \overline{AB}$	BONUS	If a = b, then a + c = b + c for all numbers a, b, and c.
The measure of the angle is 90°. 	 ∡1 and ∡2	⊥	If C is between A and B, then AC + CB = AB. 

POLYNOMIAL RUMMY

An example set of 44 cards would contain one each of the following.

$3(x^3 + 7x^2 + 10x)$
$3(x^2 + 2x)(x + 5)$
$x(x + 2)(3x + 15)$
$x(3x^2 + 21x + 30)$
$3x^3 + 21x^2 + 30x$
$3x(x + 2)(x + 5)$

$4x^2 - 4y^2$
$4(x^2 - y^2)$
$4(x + y)(x - y)$
$(2x + 2y)(2x - 2y)$
$4(x - y)(x + y)$
$(2x - 2y)(2x + 2y)$

$6(x - 5)(x + 2)$
$6(x^2 - 3x - 10)$
$6(x + 2)(x - 5)$
$6x^2 - 18x - 60$
$(3x - 15)(2x + 4)$
$(2x - 10)(3x + 6)$

$(x^2 - 4)(x^2 - 1)$
$(x + 2)(x - 2)(x^2 - 1)$
$x^4 - 5x^2 + 4$
$(x^2 - 4)(x + 1)(x - 1)$
$(x + 2)(x - 2)(x + 1)(x - 1)$
$(x^2 - 1)(x^2 - 4)$

$x(x^2 - 1) - 2(x^2 - 1)$
$(x - 2)(x^2 - 1)$
$(x - 2)(x + 1)(x - 1)$
$x^3 - 2x^2 - x + 2$

$12x^2 - 2xy - 24y^2$
$2(6x^2 - xy - 12y^2)$
$2(3x + 4y)(2x - 3y)$
$2(2x - 3y)(3x + 4y)$

$5(6x^2 - xy - y^2)$
$5(3x + y)(2x - y)$
$30x^2 - 5xy - 5y^2$
$5(2x - y)(3x + y)$

$x(x^2 - 10x + 25)$
$x(x - 5)(x - 5)$
$x(x - 5)^2$
$x^3 - 10x^2 + 25x$

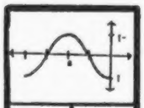
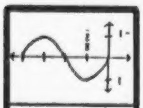
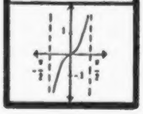
$x^2 + 6x + 8$
$(x + 4)(x + 2)$
$8 + 6x + x^2$
$(2 + x)(4 + x)$

TRIGOMINOES

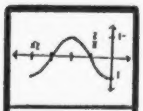
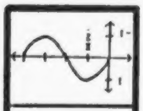
A set of 27 trigominoes has been created randomly pairing trigonometric identities.

cscθ	tanθ			
cscθ	tanθ	cscθ	$\frac{1}{\cos\theta}$	$\frac{\cos\theta}{\sin\theta}$

	cosθ	$\frac{opp}{hyp}$	$\frac{1}{\cos\theta}$	sinθ
$\frac{hyp}{opp}$	cosθ	tanθ	sinθ	sinθ

	secθ		$\frac{adj}{opp}$	tanθ
	$\frac{opp}{adj}$	$\frac{1}{\tan\theta}$	cosθ	cosθ

secθ	$\frac{opp}{hyp}$	$\frac{\sin\theta}{\cos\theta}$	cotθ	
secθ	$\frac{\cos\theta}{\sin\theta}$	cotθ	cotθ	$\frac{\sin\theta}{\cos\theta}$

			$\frac{adj}{hyp}$	$\frac{opp}{adj}$
$\frac{hyp}{opp}$	cscθ		$\frac{1}{\tan\theta}$	sinθ

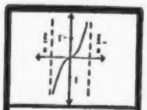
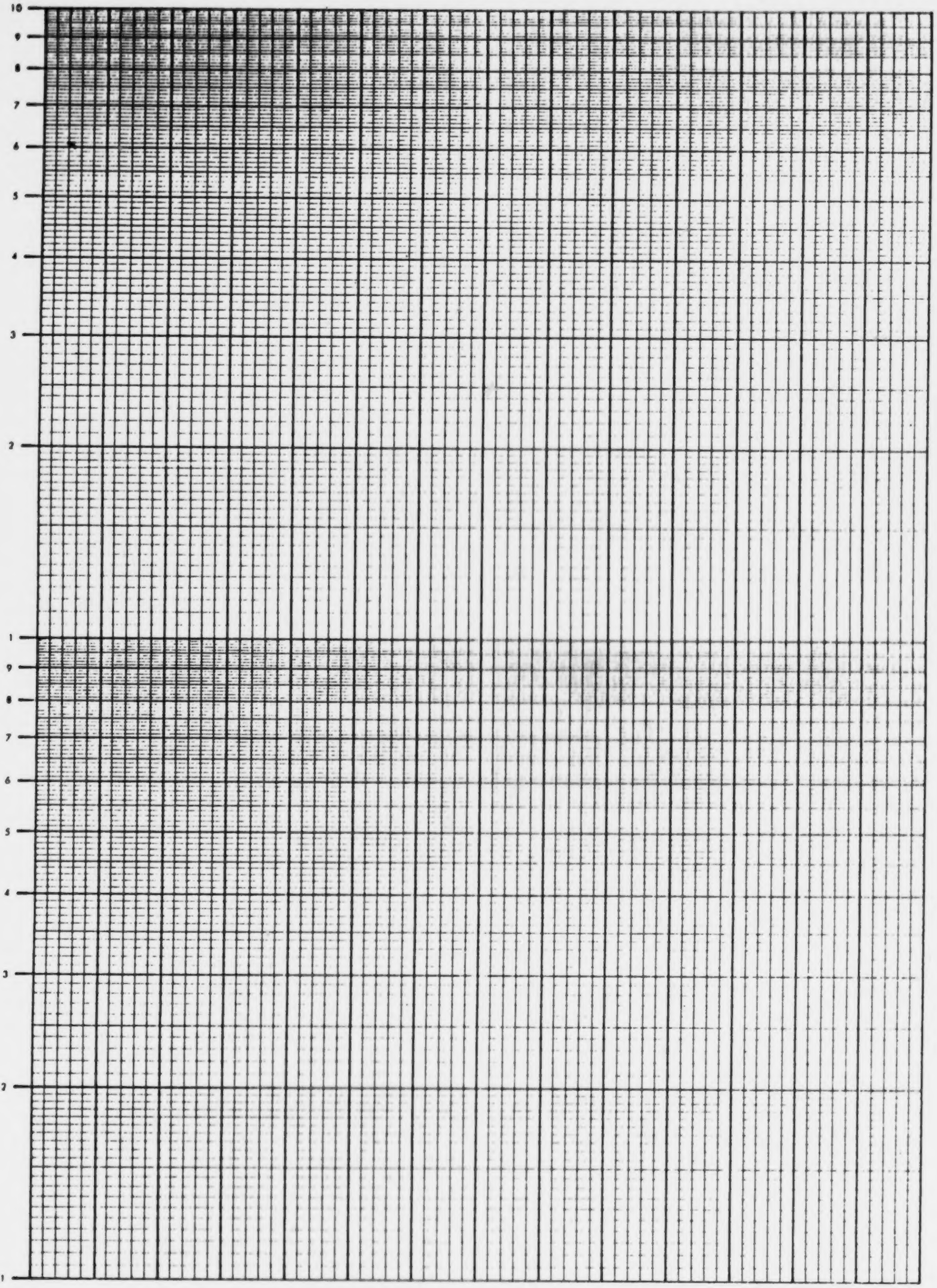
	$\frac{1}{\sin\theta}$
$\frac{adj}{hyp}$	$\frac{\cos\theta}{\sin\theta}$

TABLE OF VALUES

DEGREE MEASURE	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
RADIAN MEASURE	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$	π	$\frac{7}{6}\pi$	$\frac{5}{4}\pi$	$\frac{4}{3}\pi$	$\frac{3}{2}\pi$	$\frac{5}{3}\pi$	$\frac{7}{4}\pi$	$\frac{11}{6}\pi$	2π
SIN	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{-1}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{3}}{2}$	-1	$\frac{-\sqrt{3}}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-1}{2}$	0
COS	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{-1}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{3}}{2}$	-1	$\frac{-\sqrt{3}}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
TAN	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	—	$-\sqrt{3}$	-1	$\frac{-\sqrt{3}}{3}$	0	$\frac{-\sqrt{3}}{3}$	1	$\sqrt{3}$	—	$-\sqrt{3}$	-1	$\frac{-\sqrt{3}}{3}$	0
COT	—	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$\frac{-\sqrt{3}}{3}$	-1	$-\sqrt{3}$	—	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$\frac{-\sqrt{3}}{3}$	-1	$-\sqrt{3}$	—
SEC	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	—	-2	$-\sqrt{2}$	$\frac{-2\sqrt{3}}{3}$	-1	$\frac{-2\sqrt{3}}{3}$	$-\sqrt{2}$	-2	—	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1
CSC	—	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	—	-2	$-\sqrt{2}$	$\frac{-2\sqrt{3}}{3}$	-1	$\frac{-2\sqrt{3}}{3}$	$-\sqrt{2}$	-2	—



GEYER INSTRUCTIONAL AIDS CO., INC.

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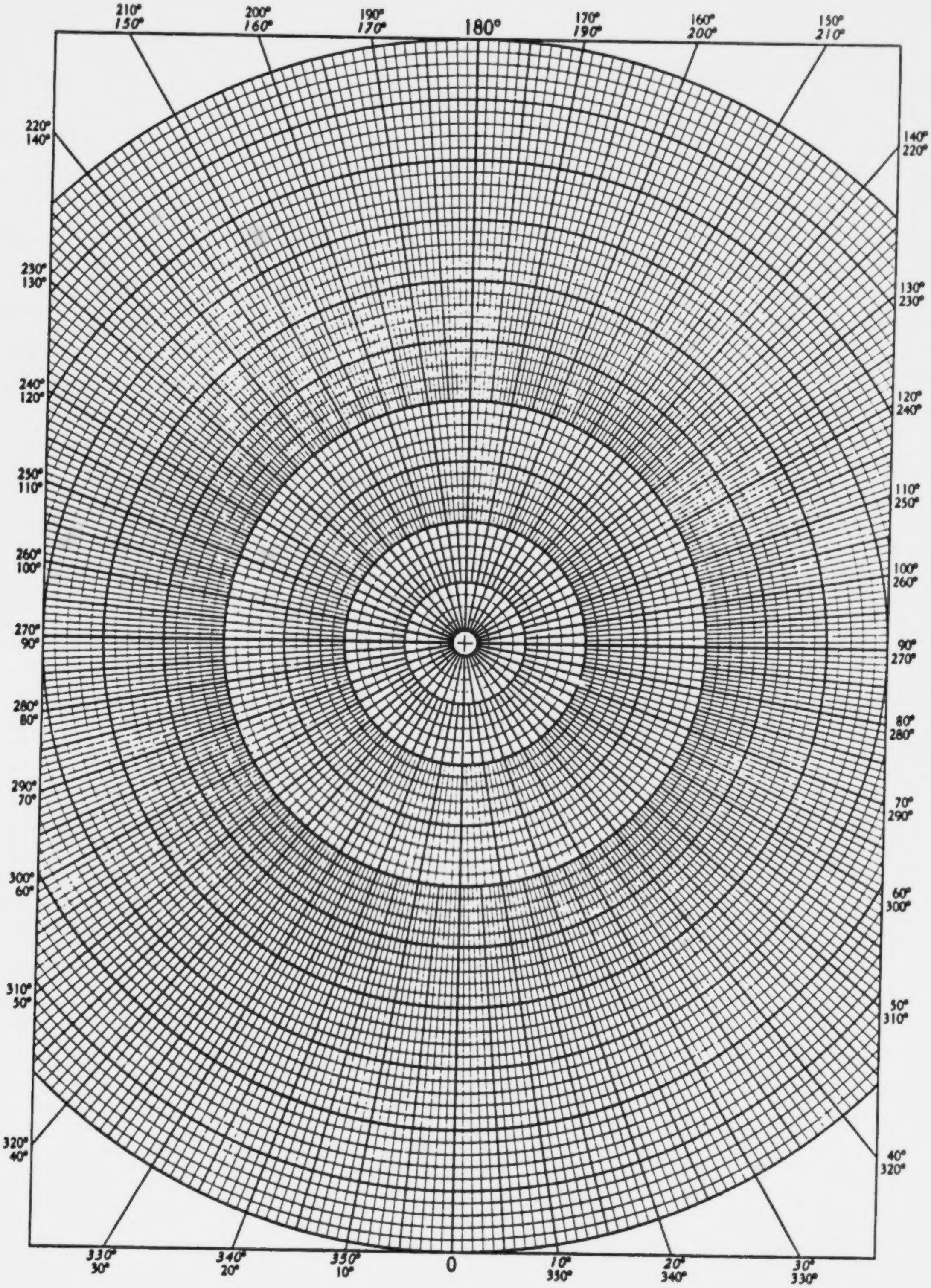
Fort Wayne, Indiana 46850



210° 150°
200° 160°
190° 170°
180°
170° 190°
160° 200°
150° 210°
220° 140°
140° 220°
230° 130°
130° 230°
240° 120°
120° 240°
250° 110°
110° 250°
260° 100°
100° 260°
270° 90°
90° 270°
280° 80°
80° 280°
290° 70°
70° 290°
300° 60°
60° 300°
310° 50°
50° 310°
320° 40°
40° 320°
330° 30°
340° 20°
350° 10°
0
10° 350°
20° 340°
30° 330°

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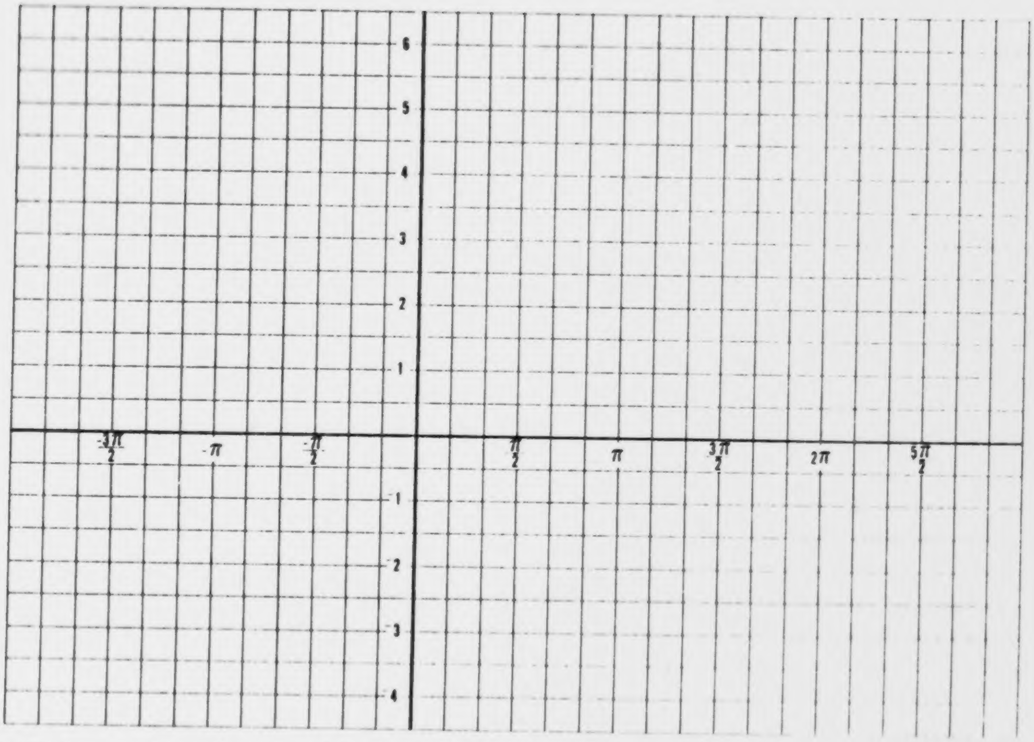

INSTRUCTIONAL AIDS CO., INC No 42 R

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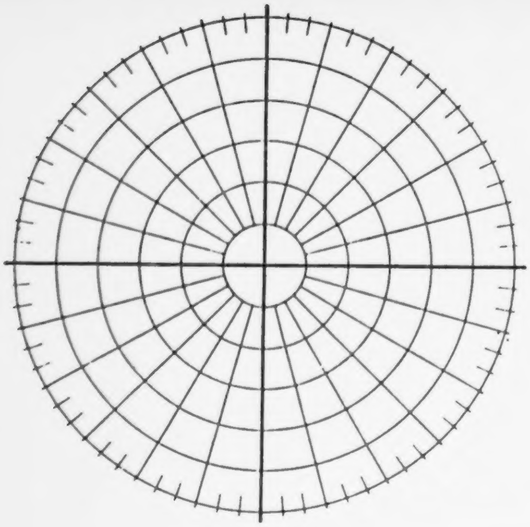

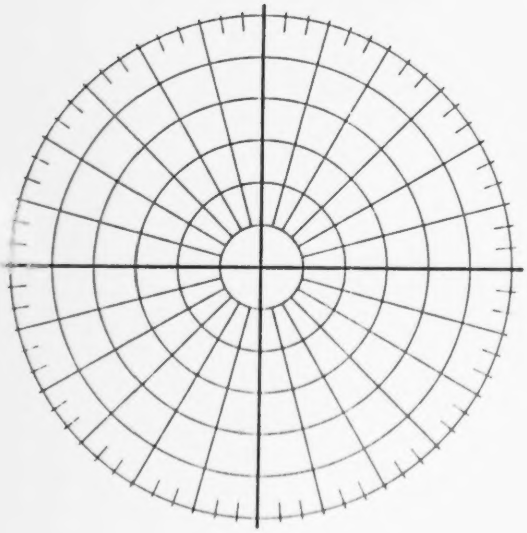

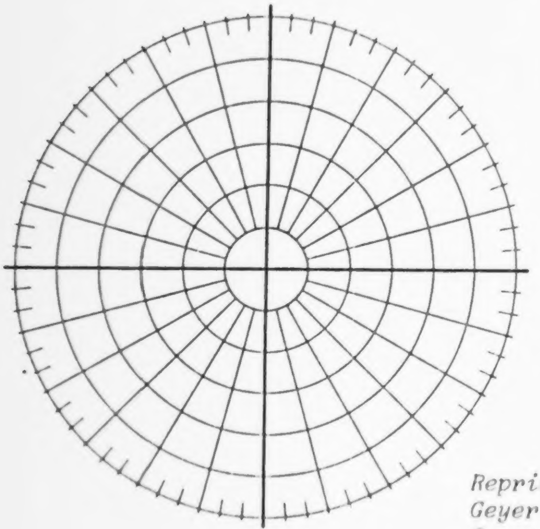

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Indiana 46850*

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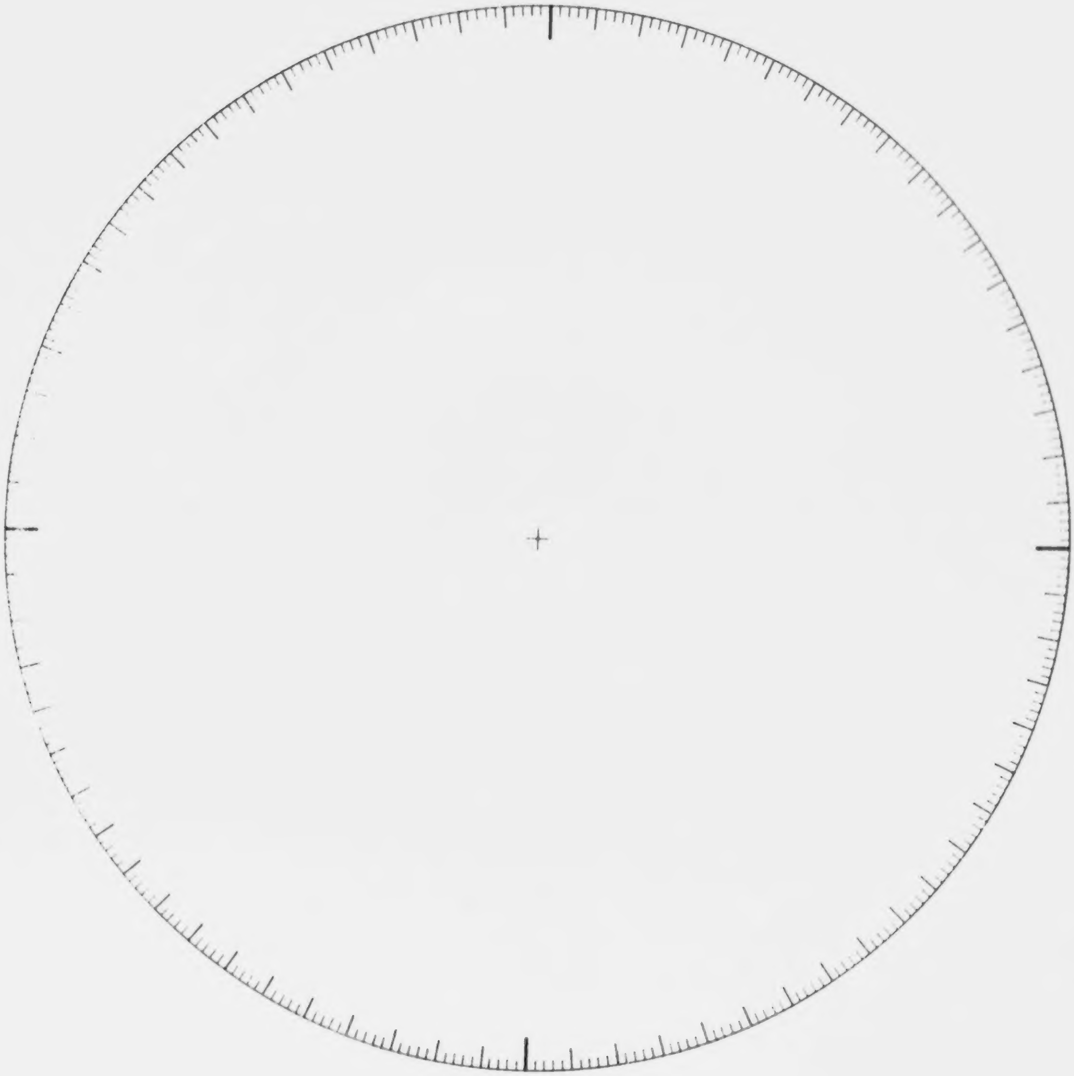

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Name ______________________________

Drawing:

Class ____________________ Date ________

Page ____________ Problem ____________

Problem:

Given:

To Prove:

Statements:

Reasons:

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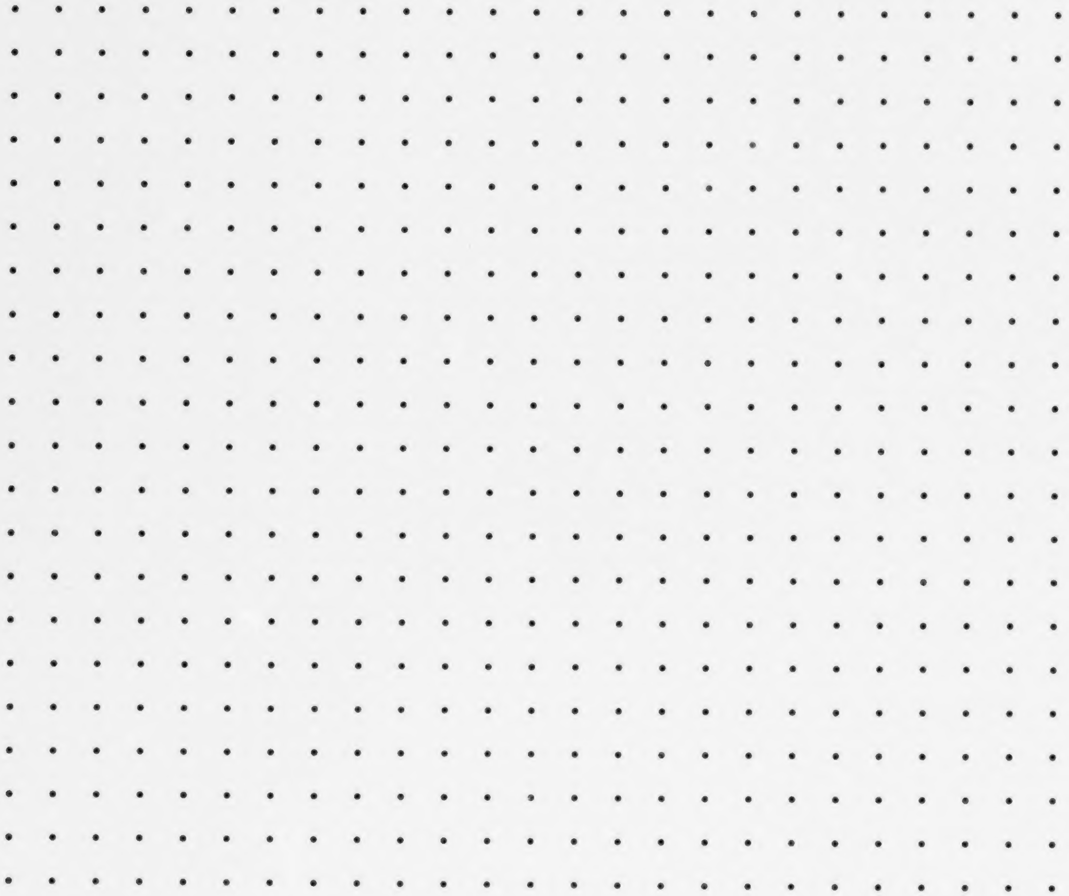
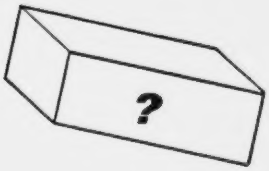
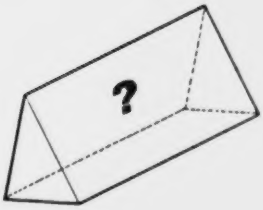
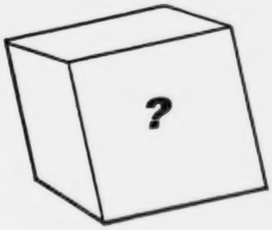
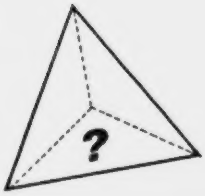
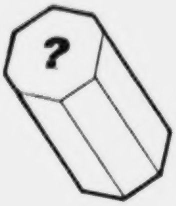










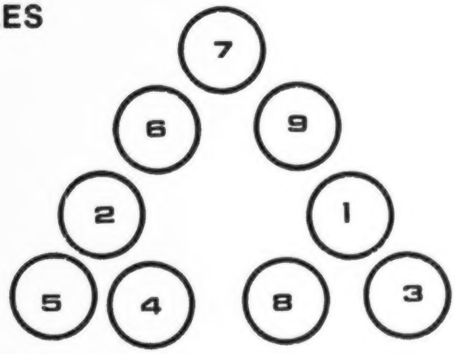


SELECTED ANSWERS

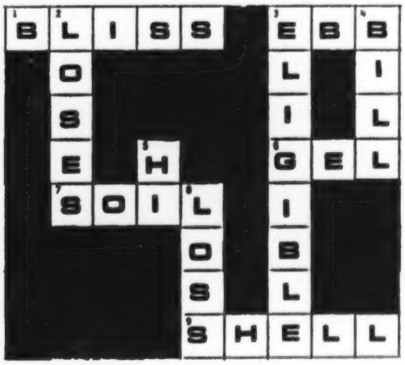
GENERAL MATH

WHOLE NUMBERS

TRIANGLE OF CIRCLES



CALCULATOR CROSSTIC



1 B	2 L	I	S	S		3 E	B	4 B
	O					L		I
	S					I		L
	E		5 H			6 G	E	L
	7 S	O	I	8 L		I		
				O		B		
				S		L		
				9 S	H	E	L	L

HURRAY ARRAY!

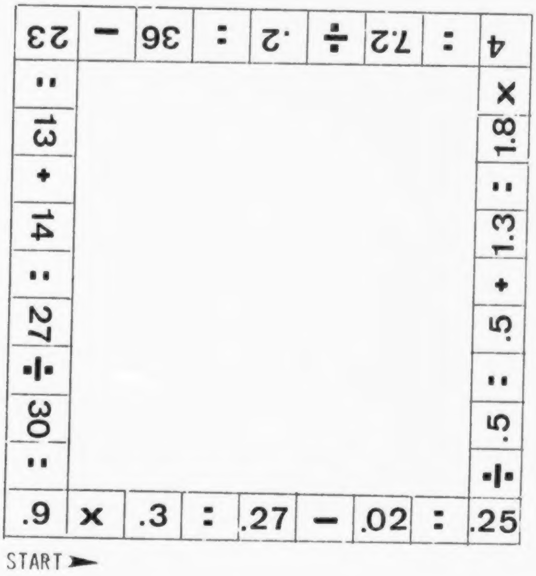
2	40	2	6	960
2	8	30	10	4800
20	2	11	2	880
3	2	2	25	300
240	1280	1320	3000	

DECIMALS

FIRST THANKSGIVING

1. 60 pounds; $89.40
2. $98.00
3. $198.00
4. 480 clams; 40 dozen; $38.00
5. $54.50
6. $47.30
7. 1440 slices; 72 loaves; $92.88
8. 30 boxes; $8.70
9. $12.45
10. $6.45
11. $4.45
12. $2.95
13. 360 ears of corn; $120.00
14. $32.50
15. $15.00

DECIMALS AROUND THE BLOCK


.9
x
.3
:
.27
−
.02
:
.25
÷
.5
:
.5
+
1.3
:
1.8
x
4
:
7.2
÷
.2
:
36
−
23
:
13
+
14
:
27
÷
30
:
START

HOW MANY STAMPS?

There are six ways show below.

5¢ stamps	8¢ stamps
7	26
15	21
23	16
31	11
39	6
47	1

FRACTIONS

TIC-TAC-TOE

$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{10}$
$\frac{3}{10}$	$\frac{3}{4}$	$\frac{1}{5}$
$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{5}$

$\frac{4}{12}$	$\frac{7}{12}$	$\frac{5}{12}$
$\frac{3}{12}$	$\frac{2}{12}$	$\frac{9}{12}$
$\frac{8}{12}$	$\frac{1}{12}$	$\frac{6}{12}$

$\frac{6}{7}$	$\frac{3}{7}$	$\frac{2}{3}$
$\frac{1}{3}$	$\frac{1}{2}$	$\frac{5}{7}$
$\frac{4}{7}$	$\frac{2}{7}$	$\frac{1}{7}$

$\frac{3}{4}$	$\frac{4}{8}$	$\frac{1}{2}$
$\frac{2}{8}$	$\frac{3}{8}$	$\frac{5}{8}$
$\frac{1}{8}$	$\frac{1}{4}$	$\frac{7}{8}$

$\frac{1}{3}$	$\frac{1}{4}$	$\frac{11}{12}$
$\frac{5}{12}$	$\frac{1}{6}$	$\frac{3}{4}$
$\frac{1}{2}$	$\frac{7}{12}$	$\frac{5}{6}$

FOOD FOR THOUGHT

8 ounces of shredded cheddar cheese
$\frac{1}{4}$ cup of chopped onion
$1\frac{1}{2}$ cups diced cooked ham
1/3 cup flour
$\frac{1}{2}$ teaspoon salt
$\frac{1}{4}$ teaspoon pepper
$\frac{1}{4}$ teaspoon caraway seed
1 cup milk
4 eggs

CLASS PRESIDENT

In the election, 340 votes were cast.

END OF THE ROPE

The length of each piece of rope is 6 3/8 feet.

BARRELS OF SAND

To store the sand, 12 1/16-ton barrels are needed. If Sam uses 1/24-ton barrels, he will need 18 barrels.
Combinations of both sizes of barrels:

1/16-ton barrels	1/24-ton barrels
2	15
4	12
6	9
8	6
10	3

HAY BALES

On the floor, 40 round bales can be placed in the shed. On the floor, 182 rectangular bales can be placed. To fill the shed, 120 round bales are needed. To fill the shed, 2,366 rectangular bales are needed.

RATIO AND PROPORTION

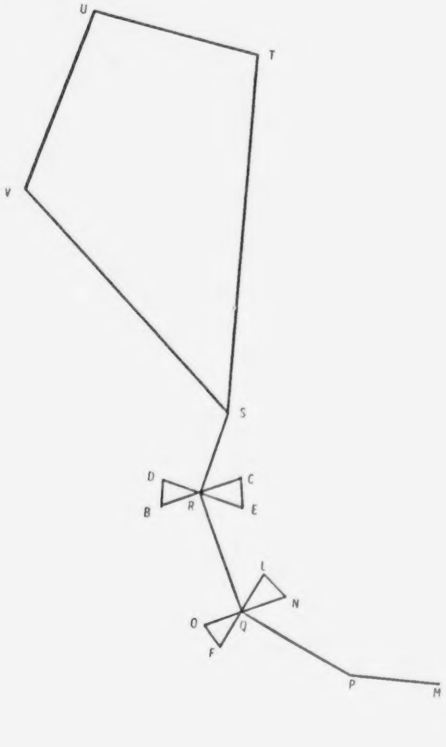
CRICKETS

One possible formula is:

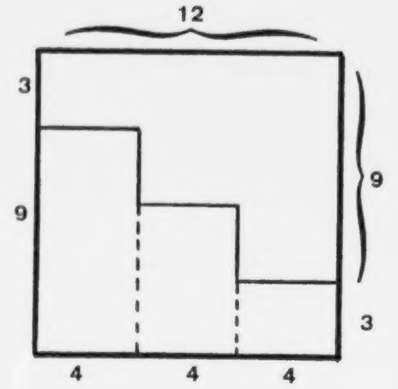
$$\text{Temperature} = \frac{\text{chirps in 1 minute}}{4} + 40$$

MEASUREMENT

FLY A KITE


U
T
V
S
D
C
B
R
E
L
N
O
Q
F
P
M

CUT A RUG



ALGEBRA I

REAL NUMBERS

INTEGER MAGIC SQUARES

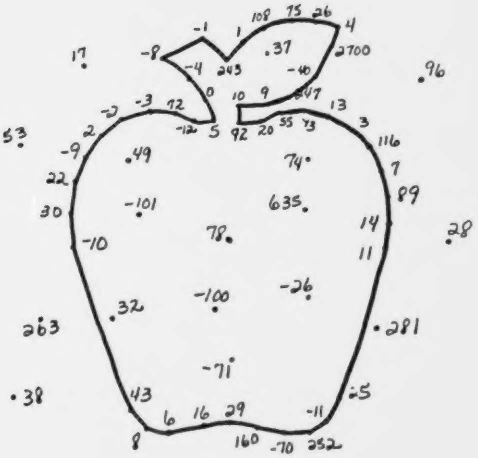
3	-4	1
-2	0	2
-1	4	-3

6	-1	4
1	3	5
2	7	0

-4	-9	-2
-3	-5	-7
-8	-1	-6

-4	-18	-8
-14	-10	-6
-12	-2	-16

SCHOOL DAZE PUZZLE


-1
1
108
75
26
4
17
-8
-4
243
.37
2700
.96
-40
0
10
9
247
13
-2
-3
72
-12
5
92
20
55
73
3
53
2
116
-9
.49
74
7
22
89
30
-101
635
14
28
-10
78
11
-26
-100
32
263
281
-71
38
43
-5
6
16
29
-11
8
160
-70
252

EVALUATING MAGIC

$\frac{w-rs}{n-x} =$ 4	$\frac{y-(r+s)}{w-3n} =$ -3	$\frac{s(w-n)}{xw-(r+s)} =$ 2
$\frac{sn-w}{n+s-r} =$ -1	$\frac{wx-s}{3n-x} =$ 1	$\frac{rn-y}{x(n-s)} =$ 3
$xy(n+s) =$ 0	$\frac{rsnwx}{w+rs} =$ 5	$\frac{sn+wx}{s-w} =$ -2

EQUATIONS

102 COOKIES

Julie ate 7 cookies on the first day.

CHOCOLATE ALGEBRA PIE

1. Cream 1/2 cup of butter with 3/4 cup of sugar.
2. Stir in 2 squares of cooled melted unsweetened chocolate.
3. Add 2 eggs, one at a time, beating at high speed 5 minutes after each addition.
4. Fold in 2 cups of whipped topping (thawed).
5. Pour into a cooled baked 9-inch pie shell. Chill until firm, about two hours, or freeze.

TAKE HOME PAY

James must work 53 1/3 hours in a week.

CODED EQUATIONS

W = 0
G = 1
J = 2
R = 3
C = 4

M = 5
N = 6
S = 7
L = 8
B = 9

OLD TEX'S GOLD

The distance from the windmill to the lake is 8 km. The gold is at the windmill.

SLOPING RAFTERS

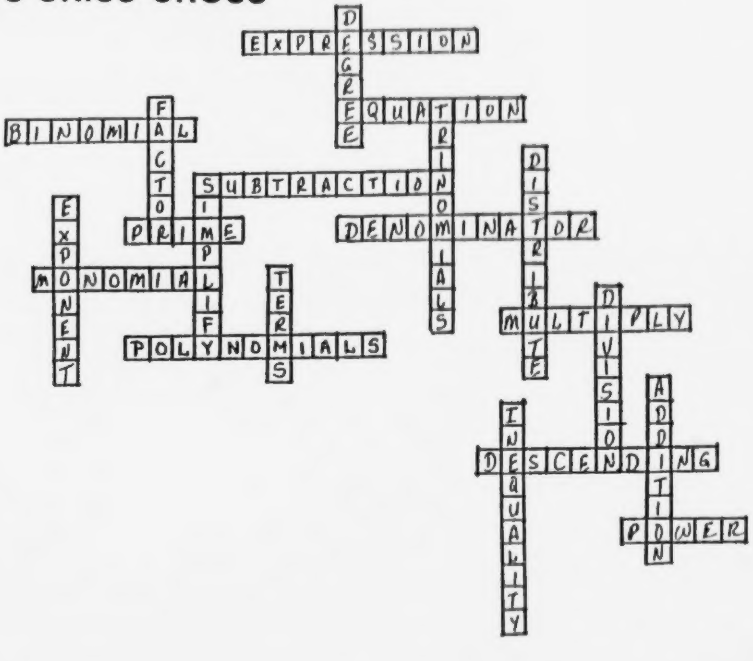
The length of the rafter is 10 ft 10 in. The length of the rafter cut with a 2-foot overhang is 12 ft 10 in.

DRIBBLES

The length and width of the ideal basketball court are 84 feet and 50 feet respectively.

POLYNOMIALS

ALGEBRAIC CRISS-CROSS


EXPRESSION
DEGREE
EQUATION
FACTOR
BINOMIAL
TRINOMIALS
SUBTRACTION
SIMPLIFY
PRIME
DENOMINATOR
DISTRIBUTIVE
EXPONENT
MONOMIAL
TERMS
POLYNOMIALS
MULTIPLY
DIVISION
ADDITION
INEQUALITY
DESCENDING
POWER

TRY A DIGIT

1.
$$\begin{array}{r} 7M + 2N \\ 5M + 4N \\ \hline 12M + 6N \end{array} \quad \text{or} \quad \begin{array}{r} 5M + 2N \\ 7M + 4N \\ \hline 12M + 6N \end{array}$$

2. $(8X + 4) - (5X - 9) = 3X + 13$

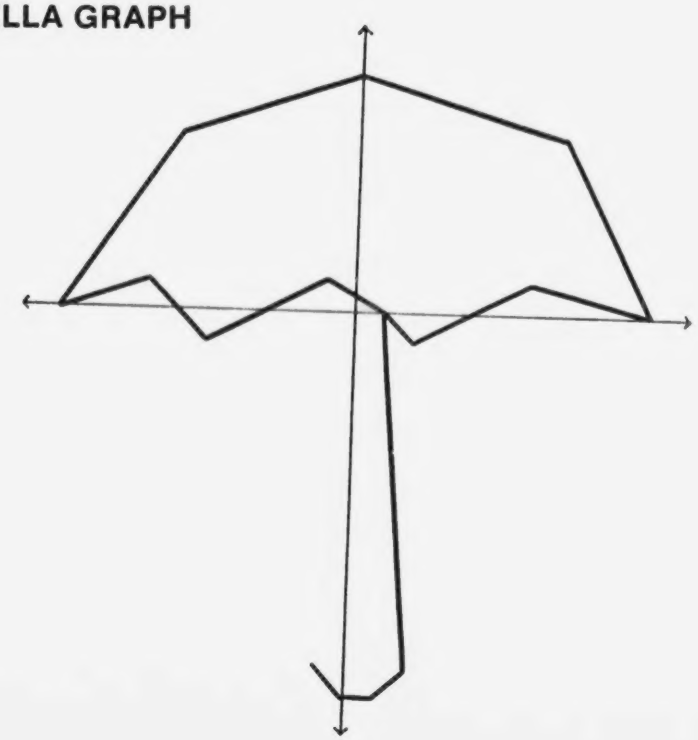
3. $(2A + 1)(A + 2) = 2A^2 + 5A + 2$

SPELLING WITH ALGEBRA

1. SMILE
2. SURPRISE
3. SPRING + FEVER

GRAPHING

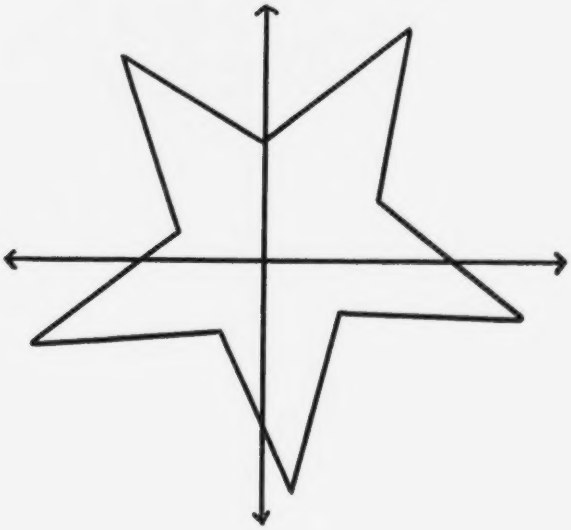
AN UMBRELLA GRAPH



SYSTEMS OF EQUATIONS

STAR SYSTEMS

1. (0, 4)
2. (-5, 7)
3. (-3, 1)
4. (-8, -3)
5. $(-1\frac{1}{2}, -2\frac{1}{2})$
6. (1, -8)
7. $(2\frac{1}{2}, -2)$
8. (9, -2)
9. (4, 2)
10. (5, 8)



GEOMETRY

PROOFS

$5 = 0$

Since $X = 5$, then $(X - 5) = 0$. Division by zero in step 5 is not possible.

SIMILAR POLYGONS

PHOTO REDUCTION

The photo length will be 15 inches.

WATER PIPELINE

The plant, P, is located 28 miles from point C.

SOLIDS

PAINTING POLYHEDRA

When each face is painted completely with a single color and no two adjacent faces have the same color, the least number of colors for each polyhedron is given below.

Tetrahedron	4 colors
Hexahedron	3 colors
Octahedron	2 colors
Dodecahedron	4 colors
Icosahedron	3 colors

ALGEBRA II

EQUATIONS

BANANA BREAD

1. Cream 1 cup of butter or margarine and 2 cups of sugar in one bowl.
2. Beat 4 eggs until light. Add eggs to the creamed mixture.
3. Mash 6 bananas thoroughly with a fork and blend with the mixture.
4. Sift 4 cups of flour, 2 teaspoons of soda and ½ teaspoon of salt together and mix quickly with the other ingredients.
5. Separate mixture into 3 equal quantities and place in loaf pans.
6. Bake for 60 minutes at 325 degrees.

PICTURE FRAMES

The choices are:

cartons of 5"x7" frames	cartons of 8"x10" frames
16	1
14	4
12	7
10	10
8	13
6	16
4	19
2	22
0	25

A TIN BOX

The piece of tin needed is 13 inches by 13 inches.

$3 = -3$

Since $X = 3$, then $(X - 3) = 0$. Division by zero in step 6 is not possible.

SYSTEMS OF OPEN SENTENCES

BASKETBALL COURT

The length and width of the ideal court are 84 feet and 50 feet respectively.

STAR SUM

$$m\measuredangle A + m\measuredangle B + m\measuredangle C + m\measuredangle D + m\measuredangle E = 180^\circ$$

BOXES OF CANDY

The maximum revenue occurs when 71 3/7 boxes of type A and 57 1/7 boxes of type B are produced.

FUNCTIONS AND RELATIONS

BOYLE'S LAW

The volume is $90 in^3$.

COMBINED VARIATION

The resistance is 5.625 ohms.

CHARTER A BUS

A group of 75 people will produce a maximum revenue of $2,812.50.

SERIES

RUNNING FOR OFFICE

On the 7th day, 729 posters will be made. On the nth day, 3^{n-1} posters will be made.

By the end of the 7th day, he will have a total of 1093 posters. By the end of the nth day, a total of $\frac{1-3^n}{-2}$ posters will be made.

DOUBLE DO IT TO IT

During the nth minute, 2^{n-1} squares will be colored. The formula for the sum of the squares colored after the nth minute is 2^n-1.

TRIGONOMETRY

RIGHT TRIANGLES

AIRPLANE FLIGHTS

1. Altitude 14.8; horizontal 31.7
2. .26

VECTORS

FLY WITH ME

North 117.9; West 92.7

Groundspeed 302.6 mph; track $7^\circ 36'$

SAILING

Distance 213.7; course N$75^\circ 45'$W

CIRCULAR FUNCTIONS

CONVERTIBLES

Radians	To Degrees
$\frac{4\pi}{5}$	135°
$\frac{10\pi}{9}$	72°
$\frac{\pi}{4}$	60°

TRIGONOMETRIC FORMULAS

MACH 2

$m = 3.9$ $\alpha = 60^\circ$

$m = 2$ $\alpha = 84^\circ$

$m = 2.63$ $\alpha = 47^\circ 10'$

FIND THE RADIO

1. a = 2.32; b = 5.28

2. The ambiguous case applies.

ANALYSIS

SEQUENCES AND SERIES

ARITHMETIC PROGRESSION

1. $t_{12} = 32$
2. $d = 16$
3. $d = 4$
4. 3, 11, 19 or 18, 11, 3

POST

0, 33 1/3, 66 2/3, 100, 133 1/3, 166 2/3, 200 233 1/3, 266 2/3, 400, 433 1/3, 466 2/3, 500, 533 1/3, 566 2/3, 600, 633 1/3, 666 2/3, 700, 733 1/3, 766 2/3, 800

GEOMETRIC PROGRESSION

1. $t_7 = 32$
2. 2, $\underline{4}$, $\underline{8}$, $\underline{16}$, 32

   2, $\underline{-4}$, $\underline{8}$, $\underline{-16}$, 32
3. $a = 3/2$
4. $r = 3$

ARITHMETIC SERIES

1. $S_{14} = 252$
2. $d = -\frac{7}{11}$
3. $k = 10$

FINITE GEOMETRIC SERIES

1. $S = 5$
2. 4, 8, 12 or 13, 8, 3

INFINITE GEOMETRIC SERIES

1. $S = 48$
2. $S = 54/5$

CONIC SECTIONS

ECCENTRICITY

1. $e = 4/5; \ \frac{y^2}{25} + \frac{x^2}{9} = 1$
2. $e = \frac{2\ 13}{13}; \ \frac{x^2}{13} + \frac{y^2}{9} = 1$

EQUATIONS

1. $xy - 18x + y - 58 = 0$
2. $x^2 + y^2 + 3x + 2 = 0$
3. $4x^2 + y^2 = 11$
4. $x^2 + 4x - 6y + 25 = 0$

COMPLEX NUMBERS

SQUARE ROOT

1. $1 + i, -1 - i$
2. $4 - 3i, -4 + 3i$
3. $5 - 2i, -5 + 2i$

POLYNOMIAL EQUATIONS

FINDING EQUATIONS

1. $X^3 - x^2 + x - 1$
2. $x^2 - 2x^3 - 5x^2 + 6x$
3. $x^3 - 8x^2 + 21x - 20$
4. $x^4 - 2x^3 - 11x^2 + 12x + 36$
5. roots are $2, -1, -2, 3/2$

APPLICATIONS OF THE DERIVATIVE

FIRE ONE

Altitude is 32,400 feet in 45 seconds,

Reaches 20,000 feet at approximately 23 seconds and 73 seconds

ENCLOSURES

320,000 sq. ft.

ROUND 2

Altitude 25,776 ft. in 40 seconds

a. Returns to the water in 80 seconds.

b. Approximately 21 and 58 seconds.

EVALUATION

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1. Circle grade level(s) taught: 6 7 8 9 10 11 12

2. Is the content of this publication appropriate for your grade level/ subject area?

   _____appropriate _____inappropriate

3. Are the suggested activities:

   _____most helpful _____helpful _____of little help

4. In your opinion, what is the major strength of this publication?

   ____

   ____

5. In your opinion, what is the major weakness of this publication?

   ____

   ____

6. Please describe other types of mathematics publications or products which would be of assistance to you.

   ____

   ____

7. In the space below or on an attached sheet please share a successful teaching/learning activity with us.

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Please return to: Mathematics Specialist
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